Part A

1

If $\log_5 120 + (x-3) - 2\log_5\left(1-5^{x-3}\right) = -\log_5\left(0.2-5^{x-4}\right)$, then x is

2 Show

$7\log\left(\dfrac{16}{15}\right) + 5\log\left(\dfrac{25}{24}\right) + 3\log\left(\dfrac{81}{80}\right)$ is equal to $\log 2$

3 Show

. The value of $\dfrac{1}{\log_2 n} + \dfrac{1}{\log_3 n} + ... + \dfrac{1}{\log_{43} n}$ is $\dfrac{1}{\log_{43!} n}$

4 Show

$\log_7 \log_7 \sqrt{7\sqrt{\left(7\sqrt{7}\right)}}$ is equal to $1 - 3\log_7 2$

5 Show

If $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$, then x is equal to $x = a^{\frac{1}{1-\log_a z}}$

6

Given that $h = \log_b 3$ and $k = \log_b 4$, express the following in terms of h and/or k.
(a) $\log_b 12$ (b) $\log_b 16$ (c) $\log_b 0.75$ [*no calculator*]

7

Consider the functions $f(x) = e^{2x+1}$ and $g(x) = \ln\sqrt{x}$.   [*calculator allowed*]

(a) (i) Find $f^{-1}(x)$ (ii) Find $g^{-1}(x)$

(b) Show that $g(f(x)) = x + \dfrac{1}{2}$

8

Solve the equation $\left(3^x\right)\left(4^{2x+1}\right) = 6^{x+2}$ and express the solution in the form $x = \dfrac{\ln p}{\ln q}$

where $p, q \in \mathbb{Z}$.   [*no calculator*]

9

Express as a single logarithm: $\frac{5}{2}\log_a x + \log_a(x+1) - \log_a\sqrt{x}$

<u>**No calculator**</u> allowed on these exercises.

1. Rewrite the expression $2\log_b 3 - 5\log_b 2 + \log_b 8$ in the form $\log_b\left(\dfrac{m}{n}\right)$ where m and n are integers.

2. Solve for x exactly: $2\log_b x - \log_b 2 = \log_b 32$

3. Given that $y = 2^x$ show that $4^{x-1} + 8^x$ is equivalent to $\dfrac{1}{4}y^2 + y^3$.

4. Solve for x exactly: $\log_2\left(x^2 - 1\right) - \log_2\left(x + 1\right) = 2$

5. Solve for x: $81^{x-2} = 27$

6. Find the exact solution(s) to the equation: $e^{2x} + e^x = 6$

7. If $y = \log_3\sqrt{3x - 4}$ express x in terms of y in the form $x = \dfrac{a^y + b}{c}$ where a, b and c are integers.

8. Solve for x exactly: $\ln(x+1) + \ln(x-1) = 1$

Part C

1. *[5 marks]*

$$2 - \log_3(x + 7) = \log_{\frac{1}{3}} 2x$$
Solve the equation .

2. *[5 marks]*

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

3

If $\log_2 4\sqrt{2} = x$, $\log_z y = 4$, and $y = 4x^2 - 2x - 6 + z$, find y.

(A) 0   (B) 2   (C) $\dfrac{5}{2}$ (D) 4   (E) 16

4. *[5 marks]*

Solve the equation $4^{x-1} = 2^x + 8$.

5. *[6 marks]*

Solve the following system of equations.

$$\log_{x+1} y = 2$$

$$\log_{y+1} x = \frac{1}{4}$$

6. *[5 marks]*

Consider $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{31} 32$. Given that $a \in \mathbb{Z}$, find the value of a.

7. *[5 marks]*

Solve the equation $8^{x-1} = 6^{3x}$. Express your answer in terms of $\ln 2$ and $\ln 3$.