Part A

1

$$
\begin{aligned}
&1-5^{x-3}>0\\
&\Rightarrow 5^{x-3}<1\\
&\Rightarrow x-3<0\\
&\Rightarrow x<3
\end{aligned}
$$

And

$$
\begin{aligned}
&0.2-5^{x-4}>0\\
&\Rightarrow 5^{x-4}<0.2\\
&\Rightarrow 5^{x-4}<5^{-1}\\
&\Rightarrow x-4<-1\\
&\Rightarrow x<3 \qquad \qquad \text{......(1)}
\end{aligned}
$$

$$\log_5 120+(x-3)-2\log_5\left(1-5^{x-3}\right)=-\log_5\left(0.2-5^{x-4}\right)$$

$$
\begin{aligned}
&\Rightarrow \log_5(24\times5)+(x-3)-2\log_5\left(1-5^{x-3}\right)=-\log_5\left(0.2-5^{x-4}\right)\\
&\Rightarrow \log_5 24+1+(x-3)-2\log_5\left(1-5^{x-3}\right)=-\log_5\left\{0.2\left(1-5^{x-3}\right)\right\}\\
&\Rightarrow \log_5 24+x-2-2\log_5\left(1-5^{x-3}\right)=-\log(0.2)-\log_5\left(1-5^{x-3}\right)\\
&\Rightarrow \log_5 24+x-2-\log_5\left(1-5^{x-3}\right)=1\\
&\Rightarrow \log_5\left\{\frac{1-5^{x-3}}{24}\right\}=x-3\\
&\Rightarrow \frac{1-5^{x-3}}{24}=5^{x-3}\\
&\Rightarrow 1=25.5^{x-3}\\
&\Rightarrow 1=5^{x-1}\\
&\Rightarrow x-1=0\\
&\Rightarrow x=1
\end{aligned}
$$

2

$$
\begin{aligned}
&7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)\\
&=7\log\left(\frac{2^4}{5\times3}\right)+5\log\left(\frac{5^2}{2^3\times3}\right)+3\log\left(\frac{3^4}{2^4\times5}\right)
\end{aligned}
$$

$$
\begin{aligned}
&=7\{4\log2-\log5-\log3\}+5\{2\log5-3\log2-\log3\}+3\{4\log3-4\log2-\log5\}\\
&=\log2
\end{aligned}
$$

3

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+....+\frac{1}{\log_{43} n}$$
$$=\log_n 2+\log_n 3+....+\log_n 43$$
$$=\log_n(2\cdot 3\cdot ...43)$$
$$=\log_n 43!$$
$$=\frac{1}{\log_{43!} n}$$

4

$$\log_7 \log_7 7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}=\log_7\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)=\log_7\left(\frac{7}{8}\right)$$
$$=1-\log_7 8$$
$$=1-3\log_7 2$$

5

$$\log_a y=\frac{1}{1-\log_a x}$$

$\therefore$ $$1-\log_a y=1-\frac{1}{1-\log_a x}$$
$$=\frac{-\log_a x}{1-\log_a x}$$

or $$\frac{1}{1-\log_a y}=\frac{1-\log_a x}{-\log_a x}$$

But $$z=a\frac{1}{1-\log_a y}$$

$\Rightarrow$ $$\log_a z=\frac{1}{1-\log_a y}=-\frac{1}{\log x}+1$$

$\Rightarrow$ $$\frac{1}{\log_a x}=1-\log_a z$$

$$\log_a x=\frac{1}{1-\log_a z}$$

$\therefore$ $$x=a^{\frac{1}{1-\log_a z}}$$

6 (a) $h+k$ (b) $2k$ (c) $h-k$

7 (a) (i) $f^{-1}(x)=\frac{1}{2}(\ln x-1)$ (ii) $g^{-1}(x)=e^{2x}$

$x=\frac{\ln 9}{\ln 8}\quad\left[\text{or }\ x=\frac{\ln 81}{\ln 64}\right]$

8

9

$\log_a\left(x^3+x^2\right)$

Part B

Answers

1. $\log_b\left(\frac{9}{4}\right)$

2. $x=8$

3. - - -

4. $x=5$

5. $x=\frac{11}{4}$

6. $x=\ln 2$

7. $x=\frac{9^y+4}{3}$

8. $x=\sqrt{e+1}$

Part C

1. *[5 marks]*

Markscheme

$\log_3\left(\frac{9}{x+7}\right) = \log_3 \frac{1}{2x}$ ***M1M1A1***

Note: Award ***M1*** for changing to single base, ***M1*** for incorporating the 2 into a log and ***A1*** for a correct equation with maximum one log expression each side.

$x + 7 = 18x$ ***M1***

$x = \frac{7}{17}$ ***A1***

2. *[5 marks]*

Markscheme

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$ ***(M1)(A1)***

$g(x) = -h(x) = 2 - \ln(x-3)$ ***M1***

Note*:* Award **M1** only if it is clear the effect of the reflection in the *x*-axis:

the expression is correct ***OR***

there is a change of signs of the previous expression ***OR***

there's a graph or an explanation making it explicit

$= \ln e^2 - \ln(x-3)$ ***M1***

$= \ln\left(\frac{e^2}{x-3}\right)$ ***A1***

3 (d) $y = 16$

4

4. *[5 marks]*

Markscheme

$2^{2x-2} = 2^x + 8$ ***(M1)***

$\frac{1}{4}2^{2x} = 2^x + 8$ ***(A1)***

$2^{2x} - 4 \times 2^x - 32 = 0$ ***A1***

$(2^x - 8)(2^x + 4) = 0$ ***(M1)***

$2^x = 8 \Rightarrow x = 3$ ***A1***

Notes: Do not award final ***A1*** if more than 1 solution is given.

5. *[6 marks]*

Markscheme

$\log_{x+1} y = 2$

$\log_{y+1} x = \frac{1}{4}$

so $(x+1)^2 = y$ ***A1***

$(y+1)^{\frac{1}{4}} = x$ ***A1***

EITHER

$x^4 - 1 = (x+1)^2$ ***M1***

$x = -1$, not possible ***R1***

$x = 1.70$, $y = 7.27$ ***A1A1***

OR

$_1(x^2 + 2x + 2)^{\frac{1}{4}} - x = 0$ ***M1***

attempt to solve or graph of LHS ***M1***

$x = 1.70$, $y = 7.27$ ***A1A1***

6. *[5 marks]*

Markscheme

$\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \ldots \times \frac{\log 32}{\log 31}$ ***M1A1***

$= \frac{\log 32}{\log 2}$ ***A1***

$= \frac{5 \log 2}{\log 2}$ ***(M1)***

$= 5$ ***A1***

hence $a = 5$

Note: Accept the above if done in a specific base *eg* $\log_2 x$.

[5 marks]

7. *[5 marks]*

Markscheme

METHOD 1

$2^{3(x-1)} = (2 \times 3)^{3x}$ ***M1***

Note: Award ***M1*** for writing in terms of 2 and 3.

$2^{3x} \times 2^{-3} = 2^{3x} \times 3^{3x}$

$2^{-3} = 3^{3x}$ ***A1***

$\ln\left(2^{-3}\right) = \ln\left(3^{3x}\right)$ ***(M1)***

$-3\ln 2 = 3x\ln 3$ ***A1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

METHOD 2

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 2^3 = 3x\ln(2 \times 3)$ ***M1A1***

$3x\ln 2 - 3\ln 2 = 3x\ln 2 + 3x\ln 3$ ***A1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

METHOD 3

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 8 = 3x\ln 6$ ***A1***

$x = \frac{\ln 8}{\ln 8 - 3\ln 6}$ ***A1***

$x = \frac{3\ln 2}{\ln\left(\frac{2^3}{6^3}\right)}$ ***M1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

[5 marks]