Logarithm

Maximum Marks: 65

PART A

1) Find the least integer n such that $7^n > 10^5$ given that $\log_{10} 343 = 2.5353$ [3M]

2) If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$, prove that $a^a \cdot b^b \cdot c^c = 1$ [4M]

3) Prove that

(a) $\dfrac{\log_a n}{\log_{ab} n} = 1 + \log_a b$ [3M]

(b) If $a^2 + b^2 = 7ab$ then $\log\left(\dfrac{a+B}{3}\right) = \dfrac{1}{2}\left[\log a + \log b\right]$ [3M]

4) Find the value of $\left(0.2\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+......\infty\right)}$ [4M]

5) Solve the following equation for x :

(a) $a^{2x}\left(a^2+1\right) = \left(a^{3x}+a^x\right)a$

(b) $3^x \cdot 8^{\frac{x}{x+2}} = 6$

(c) $4^x + 6^x = 9^x$

(d) $2^{2x+2} - 6^x - 2\times 3^{2x+2} = 0$

(e) $16^{\sin^2 x} + 16^{\cos^2 x} = 10,\ 0 \le x \le 360$

(f) $\log_{10}\left(98 + \sqrt{x^3 - x^2 - 12x + 36}\right) = 2$

(g) $\log\left(\dfrac{1}{2^x + x - 1}\right) = x\left(\log_{10} 5 - 1\right)$ [21M]

(6) If x and y are real the solve the following;

$\log_y x + \log_x y = 2$ & $x^2 + y = 12$ [3M]

(7) Solve for x and y :

$2^{x+y} = 6^y$ & $3^x = 3\times 2^{y+1}$ [3M]

(8) Solve for x and y : $\log_2 xy = 5$ & $\log_{\frac{1}{2}}\left(\dfrac{x}{y}\right) = 1$ [4M]

(9) Solve for x

$x^{\frac{2}{3}\left(\left(\log_2 x\right)^2 + \log_2 x - \frac{5}{4}\right)} = \sqrt{2}$ [4M]

PART B

(1) Solve the following system of equations.

$$\log_{x+1} y = 2$$

$$\log_{y+1} x = \frac{1}{4}$$ [4M]

(2) Solve the equation $\log_3(x+17) - 2 = \log_3 2x$ [3M]

(3) Given that $4\ln 2 - 3\ln 4 = -\ln k$, find the value of k. [3M]

(4) Solve the equation $2^{2x+2} - 10 \times 2^x + 4 = 0,\ x \in R$. [3M]