IBHL
Mathematics Function (Max Mark: 74) **By Rakesh Jha**

1

(i) Without using a calculator, solve the inequality $\frac{5x-1}{x^2+x-6} \leq 1$. [3]

(ii) Hence solve $1+\frac{1-5\ln x}{(\ln x)^2+\ln x-6} \geq 0$. [3]

2 (GDC allowed)

The curve C has equation $y=\frac{x^2}{x-2}$.

(i) Find the equation(s) of the asymptote(s) of C. [1]

(ii) Sketch the curve C, labelling the equation(s) of its asymptote(s) and coordinates of any axial intercepts and turning points. [2]

(iii) Hence find the range of values of k for which the equation $x^2 = k\left(x^2-4\right)$ has no real roots. [2]

3 (GDC allowed)

The functions f and g are defined by

$$\mathrm{f}: x \mapsto 1-2\ln(x-1) \quad , \qquad x \in \mathbb{R},\ x>1,$$

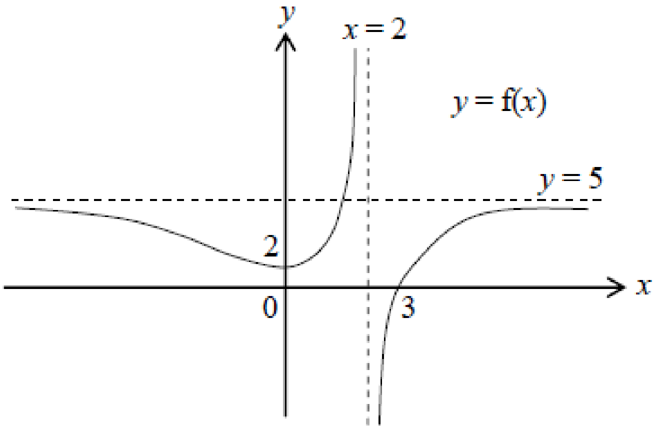
$$\mathrm{g}: x \mapsto x^2-4x+6 \quad , \qquad x \in \mathbb{R},\ x \leq 2.$$

(i) Explain why the function g^{-1} exists and express g^{-1} in a similar form. [4]

(ii) Show that the composite function fg exists. Define fg and find its range. [4]

(iii) State a sequence of transformations which transform the graph of $y=-\ln(1+x)$ to the graph of $y=\mathrm{f}(x)$. [3]

4 (GDC allowed)



The diagram shows the graph of $y = \mathrm{f}(x)$ which has a minimum point at (0, 2) and asymptotes $x = 2$ and $y = 5$.

On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y^2 = \mathrm{f}(x)$, [3]

5 (GDC allowed)

The function f is defined by

$$\mathrm{f} : x \mapsto x^2 - 2x + 2,\ 0 \le x \le 1.$$

(i) Sketch the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{ff}^{-1}(x)$ on a single diagram, indicating clearly the domains of the respective functions. [3]

(ii) Find $\mathrm{f}^{-1}(x)$. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto \frac{x + a}{x + 1},\ 0 \le x \le 3,$$

where a is a constant and $a > 1$.

(iii) Show that the composite function gf exists and find, in exact form, the range of gf. [4]

(iv) Given that $\mathrm{h}'(x) = \mathrm{gf}(x)$, show that h is an increasing function for $0 \le x \le 3$. [3]

6

Without using a graphing calculator, solve the inequality $\dfrac{4x^2 + 3x + 4}{1 - x} > 1$. [4]

7 (GDC allowed)

Functions f and g are defined by

$$f : x \to x^2 + 6x - 9, \quad \text{for } x \in \mathbb{R}, x \le 0,$$
$$g : x \to 2 - \sqrt{3x - 1}, \quad \text{for } x \in \mathbb{R}, x \ge k.$$

(i) Sketch the graph of f and state its range. [2]

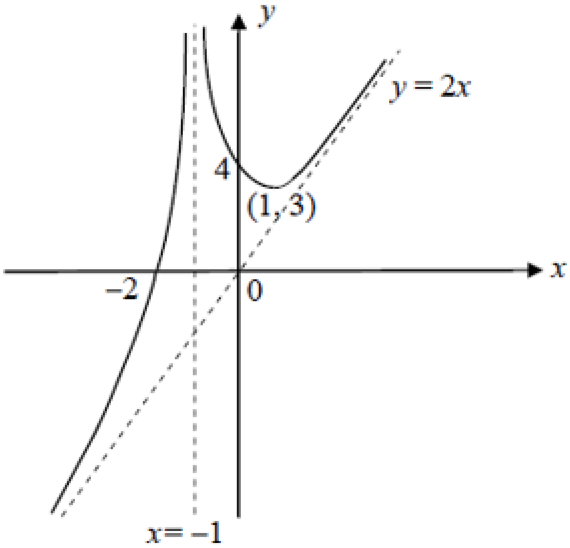
(ii) Explain why the inverse function f^{-1} does not exist. [1]

(iii) State a maximal domain of f for which the inverse function exists. [1]

(iv) By considering the graph of g, find the least value of k for which the composite function fg exists. [2]

8 (GDC allowed)

(a) The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum point at (1, 3) and intersects the axes at $x = -2$ and $y = 4$. The equations of the asymptotes are $y = 2x$ and $x = -1$.



On separate diagrams, sketch the graphs of

$y = \dfrac{1}{\mathrm{f}(x)}$, [2]

$y^2 = \mathrm{f}(x)$. [2]

In each case, give if possible, the equations of the asymptotes, the coordinates of the turning points and the coordinates of the points where the graph crosses the x- and y-axes.

(b) A curve undergoes the transformations A, B and C in succession:

A: a translation of 2 units in the negative direction of the x-axis.

B: a scaling parallel to the x-axis by a factor 2.

C: a translation of 1 unit in the positive direction of the y-axis.

The equation of the resulting curve is $y = 2x + 9 - \dfrac{2}{x+4}$.

Find the equation of the curve before the three transformations were effected.[3]

9 (GDC allowed)

The functions f, g and h are defined by

$$\mathrm{f}: x \mapsto |x-2|,\ \ x \in \mathbb{R},\ x < k,$$
$$\mathrm{g}: x \mapsto \frac{1}{x^2},\ \ x \in \mathbb{R},\ x \neq 0,$$
$$\mathrm{h}: x \mapsto \frac{1}{x^2-4x+4},\ \ x \in \mathbb{R},\ x < k.$$

State the largest value of k such that f^{-1} exists. [1]

Use this value of k for the following parts.

(i) Find f^{-1} in a similar form. Hence find the range of values of x such that $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [4]

(ii) Determine the range of values of x for which $\mathrm{f}(x) \leq \mathrm{g}(x)$, giving your answers in exact form. [4]

(iii) Show that the composite function gf exists and detemine if the functions h and gf are equal. [2]

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(a) Without using a graphic calculator, find the set of values of x such that

$$\frac{1}{2-x-x^2} \leq \frac{4}{9}.$$ [4]

(b) The functions f and g are defined by

$$\mathrm{f}: x \mapsto \sqrt{x-1} \text{ for } x \geq 1$$
$$\mathrm{g}: x \mapsto 4x^2+4x+2 \text{ for } -3 \leq x \leq 0$$

(i) Show that the composite function fg exists; [2]

(ii) Find the composite function fg in a similar form and write down its range. [3]