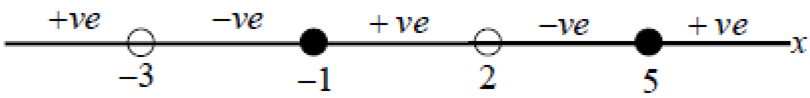
4 (i)

$$\frac{5x-1}{x^2+x-6} \le 1$$

$$\frac{x^2+x-6-(5x-1)}{x^2+x-6} \ge 0$$

$$\frac{x^2-4x-5}{x^2+x-6} \ge 0$$

$$\frac{(x+1)(x-5)}{(x+3)(x-2)} \ge 0$$



Hence $x < -3$, $-1 \le x < 2$, or $x \ge 5$.

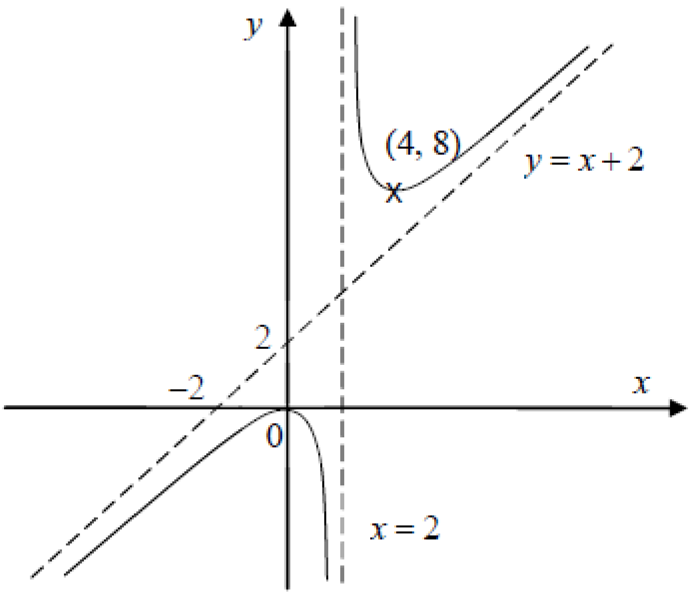
(ii)

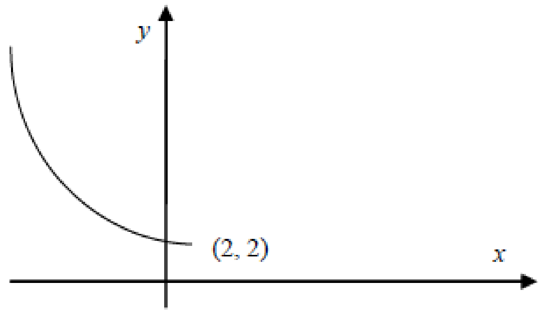
$$1+\frac{1-5\ln x}{(\ln x)^2+\ln x-6} \ge 0 \quad \Leftrightarrow \quad \frac{5\ln x-1}{(\ln x)^2+\ln x-6} \le 1$$

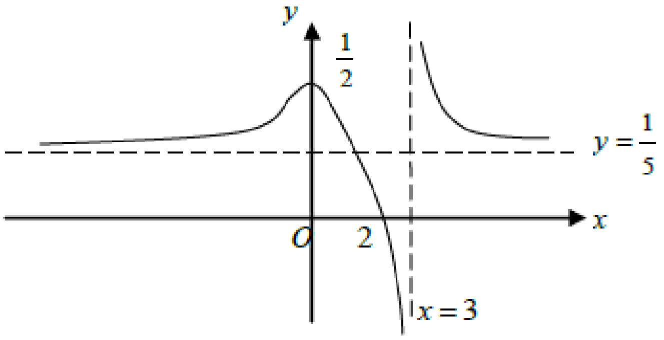
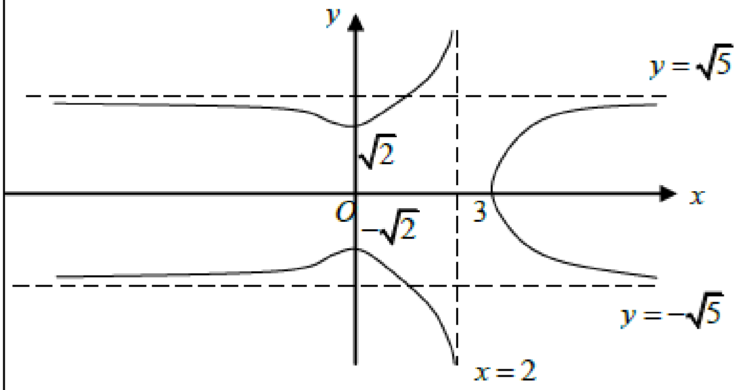
$\dfrac{5y-1}{y^2+y-6} \le 1$, where $y = \ln x$

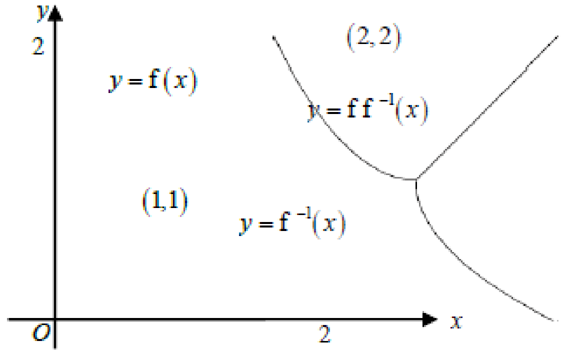
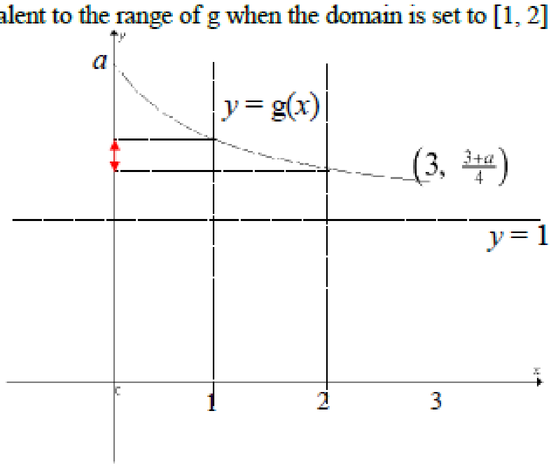
Hence $\ln x < -3$, $-1 \le \ln x < 2$, or $\ln x \ge 5$.

i.e. $0 < x < e^{-3}$, $e^{-1} \le x < e^{2}$, or $x \ge e^{5}$.

12 (i)	$y=\frac{x^2}{x-2}=x+2+\frac{4}{x-2}$ Asymptotes: $y=x+2$, $x=2$
(ii)	
(iii)	$x^2=k\left(x^2-4\right)$ $\frac{x^2}{x-2}=k\left(x+2\right)$ For all $k\in\mathbb{R}$, $y=k\left(x+2\right)$ cuts $\left(-2,0\right)$. No real roots, hence $y=\frac{x^2}{x-2}$ does not intersect $y=k\left(x+2\right)$. Thus $0<k\leq 1$.

Qn	Solution
1 **(i)**	Graph of $y = \mathrm{g}(x), x \le 2$  Any horizontal line $y = k, k \in \mathbb{R}$ cuts the graph of $y = \mathrm{g}(x), x \le 2$ at most once. Hence g is one-one. Thus g^{-1} exists. Let $y = x^2 - 4x + 6$ $y = (x-2)^2 + 2$ $x - 2 = \pm\sqrt{y-2}$ $x = 2 - \sqrt{y-2}$ (rej. $\sqrt{y-2}$ since $x \le 2$) $\mathrm{g}^{-1} : x \mapsto 2 - \sqrt{x-2}, x \ge 2$
(ii)	$R_{\mathrm{g}} = [2, \infty) \subseteq (1, \infty) = D_{\mathrm{f}}$ Hence fg exists. $\mathrm{fg}(x) = 1 - 2\ln\left(x^2 - 4x + 5\right),\ x \le 2$ $R_{\mathrm{fg}} = (-\infty, 1]$
(iii)	Translation of 2 units in the positive x-direction Scaling parallel to the y-axis by a factor of 2, followed by a translation of 1 unit in the positive y-direction. <u>OR</u> Translation of 0.5 unit in the positive y-direction, followed by scaling parallel to the y-axis by a factor of 2.

6i)	y $\frac{1}{2}$ $y=\frac{1}{5}$ x O 2 $x=3$
ii)	y $y=\sqrt{5}$ $\sqrt{2}$ x O 3 $-\sqrt{2}$ $y=-\sqrt{5}$ $x=2$

3(i)	
(ii)	Let $y=\mathrm{f}(x)=x^2-2x+2$ $y=(x-1)^2+1$ $(x-1)^2=y-1$ $x-1=\pm\sqrt{y-1}$ $x=1\pm\sqrt{y-1}$ Since $0\le x\le 1$ from the domain of f, $x=1-\sqrt{y-1}$ So $\mathrm{f}^{-1}(x)=1-\sqrt{x-1}$
(iii)	$R_\mathrm{f}=[1,\ 2]$ $D_\mathrm{g}=[0,\ 3]$ Since $R_\mathrm{f}\subseteq D_\mathrm{g}$, the composite function gf exists. The range of gf is equivalent to the range of g when the domain is set to [1, 2]  $\mathrm{g}(1)=\dfrac{1+a}{2}$ and $\mathrm{g}(2)=\dfrac{2+a}{3}$ So $R_{\mathrm{gf}}=\left[\dfrac{2+a}{3},\ \dfrac{1+a}{2}\right]$

(iv)	$h'(x) = gf(x)$ $= \dfrac{x^2-2x+2+a}{x^2-2x+2+1}$ $= \dfrac{(x-1)^2+1+a}{(x-1)^2+2}$ Since $(x-1)^2 \geq 0$ for all $x \in \mathbb{R}$, we have both $(x-1)^2+2>0$ and $(x-1)^2+1+a>0$ for all $x \in \mathbb{R}$ and $a>1$. So $h'(x)>0$ for all $x \in \mathbb{R}$, and in particular for $0 \leq x \leq 3$. Hence, h is an increasing function for $0 \leq x \leq 3$.

6

$$\frac{4x^2+3x+4}{1-x} > 1$$

$$\frac{4x^2+4x+3}{1-x} > 0$$

$$\frac{(2x+1)^2+2}{1-x} > 0$$

Since $(2x+1)^2+2>0 \quad \forall\, x \in \mathbb{R}$,

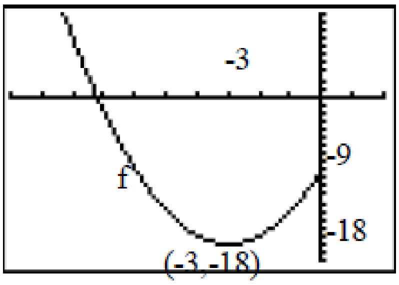
$$1-x>0$$

$$\Rightarrow x<1$$

7

$f(x)=x^2+6x-9 \;=(x+3)^2-18$

$R_f=[-18,\infty)$



(ii) Any horizontal line $y=k,\;-18<k\le -9$ cuts the graph of f at 2 points, hence f is not one-one function. f^{-1} does not exist.

(iii) Maximal domain of f for which f^{-1} exists $=(-\infty,-3]$

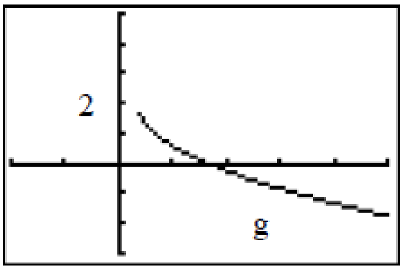
(iv) fg exists if $R_g \subset D_f$. i.e. $R_g \subset (-\infty,0]$

Let $g(x)=0$, $\quad 2-\sqrt{3x-1}=0$

$\Rightarrow 3x-1=4$

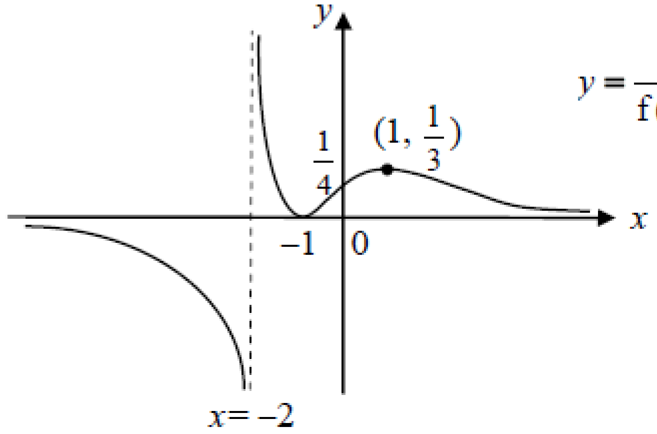
$\Rightarrow x=\dfrac{5}{3}$

Least value of $k=\dfrac{5}{3}$

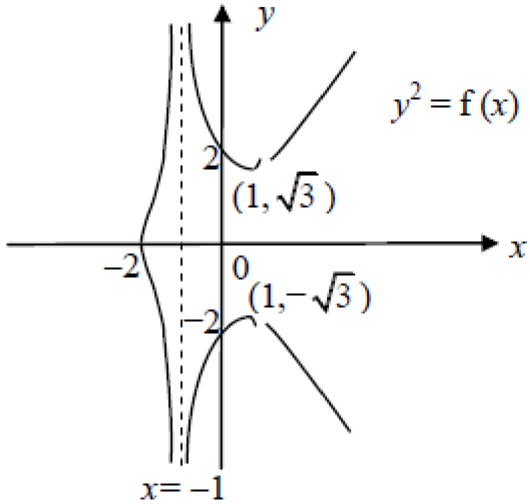


8

(ii)



(iii)



(b) $y = 2x + 9 - \frac{2}{x+4}$

↓ replace y by $y + 1$

$y = 2x + 8 - \frac{2}{x+4}$

↓ replace x by $2x$

$y = 2(2x) + 8 - \frac{2}{2x+4} = 4x + 8 - \frac{1}{x+2}$

↓ replace x by $(x-2)$

$y = 4(x-2) + 8 - \frac{1}{(x-2)+2} = 4x - \frac{1}{x}$

The equation of the curve before the transformations were effected is $y = 4x - \frac{1}{x}$

Largest $k = 2$

(i) Since $x < 2$, $f(x) = 2 - x$

Let $y = 2 - x \Rightarrow x = 2 - y$

Alternative solution:

Let $y = |x - 2|$

$y = x - 2$ or $y = -(x-2)$

$x = y + 2$ (rejected since $x < 2$) or $x = -y + 2$

$\therefore f^{-1} : x \mapsto 2 - x, \quad x > 0$

$D_f = (-\infty, 2)$ and $D_{f^{-1}} = (0, \infty)$.

For $f(x) = f^{-1}(x)$, $0 < x < 2$

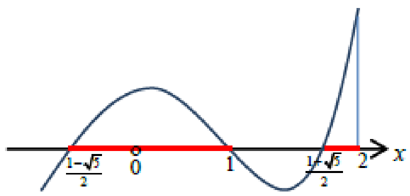
(ii) **Method 1: Algebraic Method**

$-x + 2 \le \dfrac{1}{x^2}$

$x^3 - 2x^2 + 1 \ge 0$

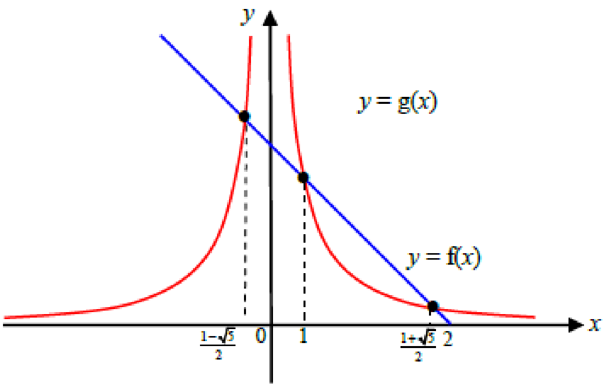
$(x-1)(x^2 - x - 1) \ge 0$ by long division

$(x-1)\left(x - \left(\dfrac{1-\sqrt{5}}{2}\right)\right)\left(x - \left(\dfrac{1+\sqrt{5}}{2}\right)\right) \ge 0$



$\dfrac{1-\sqrt{5}}{2} \le x < 0$ or $0 < x \le 1$ or $\dfrac{1+\sqrt{5}}{2} \le x < 2$

Method 2: Graphical Method



To find the intersection points:

$-x + 2 = \dfrac{1}{x^2}$

$x^3 - 2x^2 + 1 = 0$

$(x-1)(x^2 - x - 1) = 0$

$x = 1$ or $x = \dfrac{1 \pm \sqrt{5}}{2}$

From the graph, for $f(x) \le g(x)$,

$\frac{1-\sqrt{5}}{2} \le x < 0$ or $0 < x \le 1$ or $\frac{1+\sqrt{5}}{2} \le x < 2$

(iii) $R_f = (0, \infty)$, $D_g = \mathbb{R} \setminus \{0\}$

Since $R_f \subset D_g$, gf exists.

Since $D_h = (-\infty, 2) = D_{gf} = D_f = (-\infty, 2)$,

$\therefore$ $h(x) = gf(x)$

10

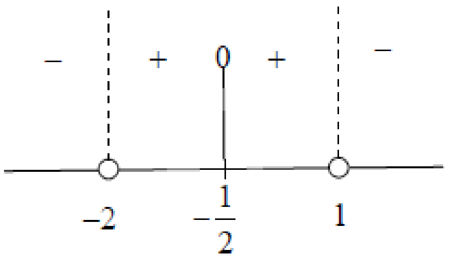
(a) $\frac{4}{9}-\frac{1}{2-x-x^2}\geq 0$

$\frac{4(2-x-x^2)-9}{2-x-x^2}\geq 0$

$\frac{-(4x^2+4x+1)}{2-x-x^2}\geq 0$

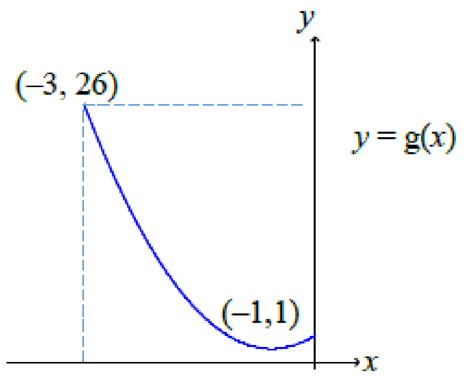
$\frac{(2x+1)^2}{2-x-x^2}\leq 0$

$\Rightarrow \frac{(2x+1)^2}{(2+x)(1-x)}\leq 0$



Using sign test, $x<-2$ or $x=-\frac{1}{2}$ or $x>1$.

(b) (i)



From the GC, $R_g=[1,26]$

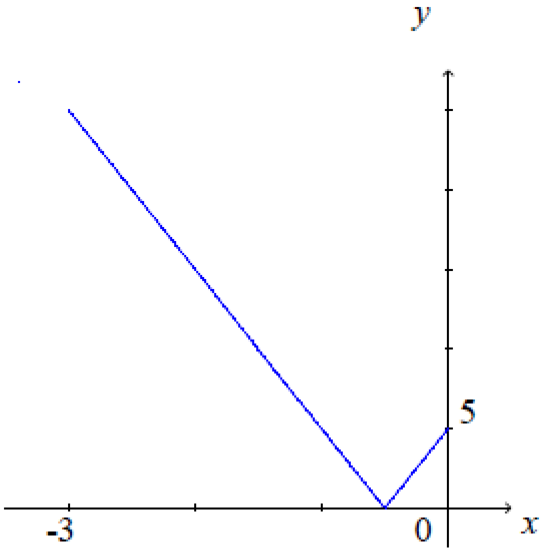
Since $R_g \subseteq D_f=[1,\infty)$, fg exists.

(ii) $fg(x)=\sqrt{4x^2+4x+1}=\sqrt{(2x+1)^2}=|2x+1|$

$fg: x \mapsto |2x+1|,\ -3\leq x\leq 0$

Or $fg: x\mapsto \begin{cases} -(2x+1), & -3\leq x<-\frac{1}{2} \\ 2x+1, & -\frac{1}{2}\leq x\leq 0 \end{cases}$

From GC,



$\therefore\ R_{fg} = [0, 5]$