◆ SOLUTION KEY ◆

Test: Functions - Basics

Show your working clearly in the space provided.

Part I - No calculator – Qs 1-6 **Total marks on test: 75**

1. Consider the quadratic function $g(x) = 3x^2 + 12x + 8$.

(a) Express $g(x)$ in the form $g(x) = a(x-h)^2 + k$. State the values of a, h and k. **[4 marks]**

$g(x) = 3(x^2 + 4x + \_\_) + 8 + \_\_$

$= 3(x^2 + 4x + 4) + 8 - 12$ cancel to zero

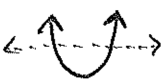
$g(x) = 3(x+2)^2 - 4 \implies a = 3,\ h = -2,\ k = -4$

(b) State the domain and range for $g(x)$. **[4 marks]**

domain: $x \in \mathbb{R}$ range: $y \geq -4$

(c) Briefly explain why the inverse of $g(x)$ is not a function. **[2 marks]**

$g(x)$ is not a one-to-one function; only one-to-one functions have inverses that are functions. OR



the graph of $g(x)$ is a parabola and does not pass the horizontal line test

(d) Restrict the domain of $g(x)$ in such a way that the domain is as large as possible but so that the inverse of $g(x)$ will be a function. State this 'new' restricted domain for $g(x)$. **[2 marks]**

'new' domain: $x \geq -2$ OR $x \leq -2$

(e) For $g(x)$ having the domain stated in (d), find $g^{-1}(x)$. **[3 marks]**

$y = 3(x+2)^2 - 4$

switch domain & range

$x = 3(y+2)^2 - 4$

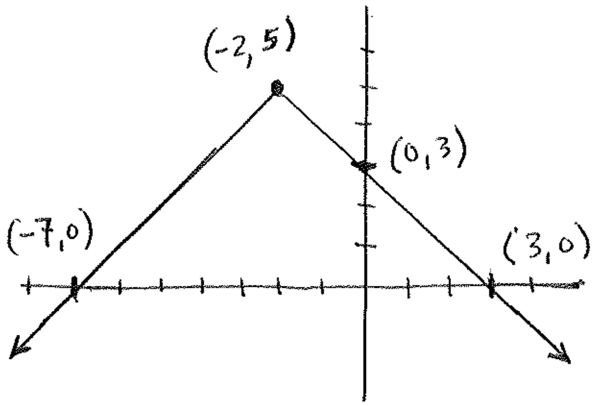
solve for y

$(y+2)^2 = \frac{x+4}{3} \implies y + 2 = \pm\sqrt{\frac{x+4}{3}}$

thus, $g^{-1}(x) = -2 + \sqrt{\frac{x+4}{3}}$

[$g^{-1}(x) = -2 - \sqrt{\frac{x+4}{3}}$ if domain of $g(x)$ is $x \leq -2$]

2. Draw an accurate sketch of the absolute value function $y = -|x+2| + 5$. Clearly label (giving coordinates) the 'vertex' of the graph and any x-intercepts or y-intercepts. **[5 marks]**



3. State the domain and range for each function.

(a) $f(x)=\sqrt{4-x}$ **[3 marks]**

domain $x \leq 4$

range $y \geq 0$

(b) $h(x)=10^{x-3}$ **[3 marks]**

domain $x \in \mathbb{R}$

range $y > 0$

(c) $g(x)=\frac{5}{x+5}$ **[3 marks]**

domain $x \in \mathbb{R},\ x \neq -5$

range $y \in \mathbb{R},\ y \neq 0$

4. Let $f(x)=\frac{1}{x+1}$, $x \neq -1$ and $g(x)=\frac{x}{3}-1$

If $h=g \circ f$, find:

(a) $h(x)$ and express it as a single simplified fraction. **[3 marks]**

(b) $h^{-1}(x)$ and express it as a single simplified fraction . **[3 marks]**

(a) $h(x)=g(f(x))=\dfrac{\frac{1}{x+1}}{3}-1=\dfrac{1}{3(x+1)}-1=\dfrac{1}{3x+3}-\dfrac{3x+3}{3x+3}=\dfrac{1-3x-3}{3x+3}$

$h(x)=\dfrac{-3x-2}{3x+3}$

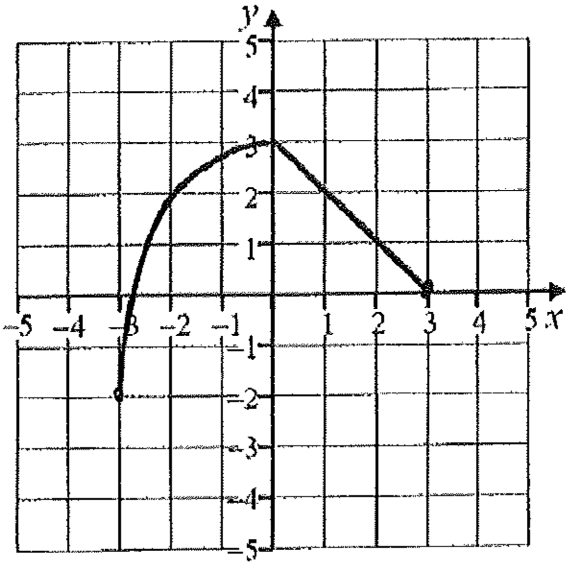
(b) $y=\dfrac{-3x-2}{3x+3}$ switch domain + range $x=\dfrac{-3y-2}{3y+3}$ solve for y (independent variable)

$x(3y+3)=-3y-2 \rightarrow 3xy+3x=-3y-2 \rightarrow 3xy+3y=-3x-2$

$y(3x+3)=-3x-2 \rightarrow y=\dfrac{-3x-2}{3x+3}$ $h^{-1}(x)=\dfrac{-3x-2}{3x+3}$

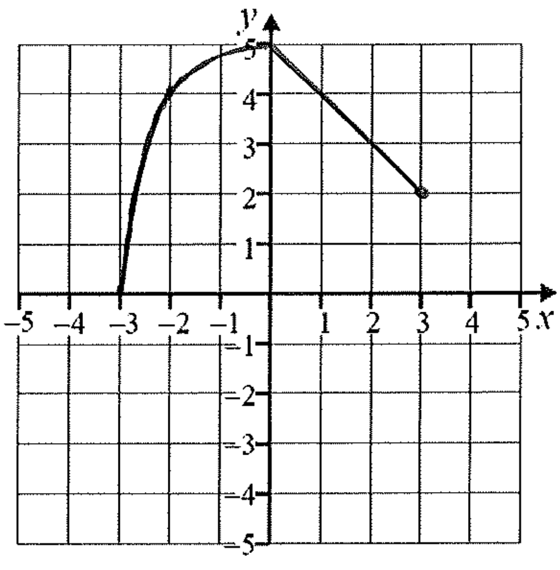
* Note: h(x) is the inverse of itself; i.e. it's 'self inverse'

5. The diagram shows a sketch of the graph of $y = f(x)$, $-3 \le x \le 3$. **[3 marks each]**

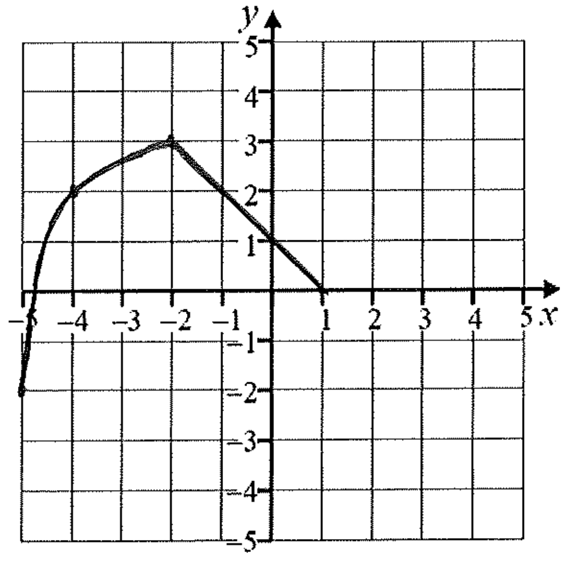


Sketch each of the graphs with the following equations.

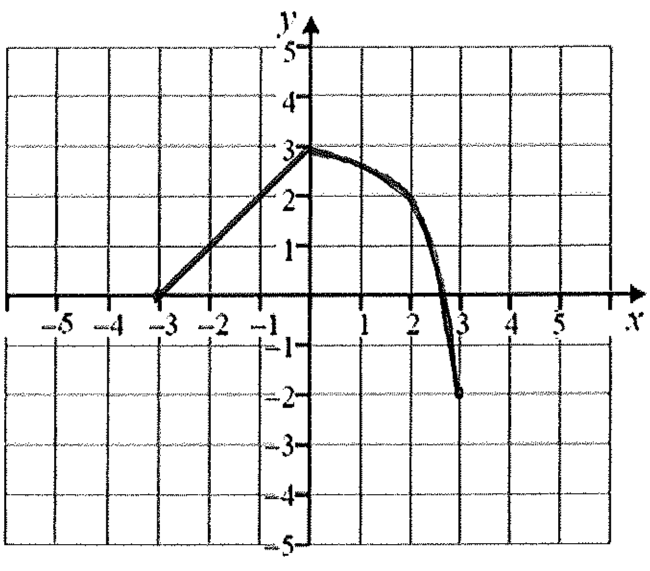
(a) $y = f(x) + 2$



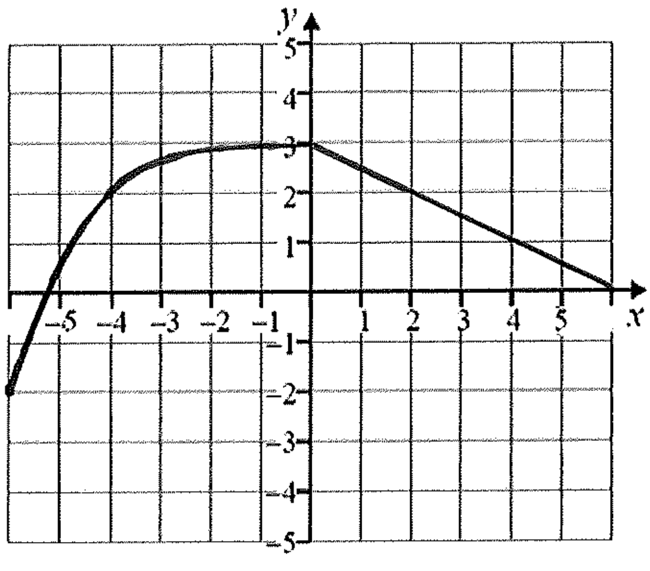
(b) $y = f(x+2)$



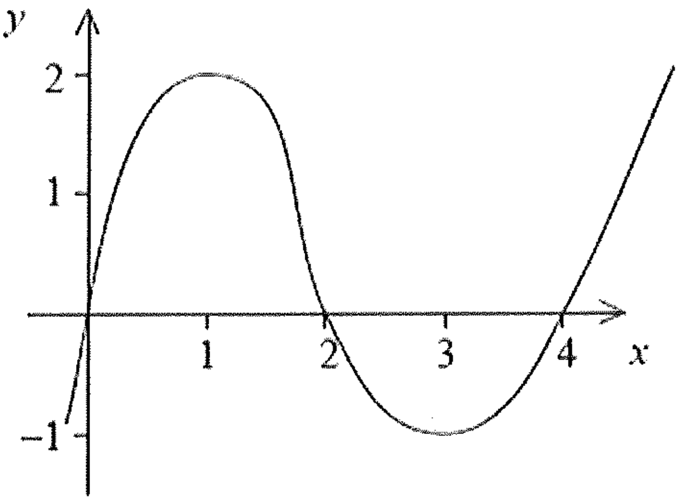
(c) $y = f(-x)$



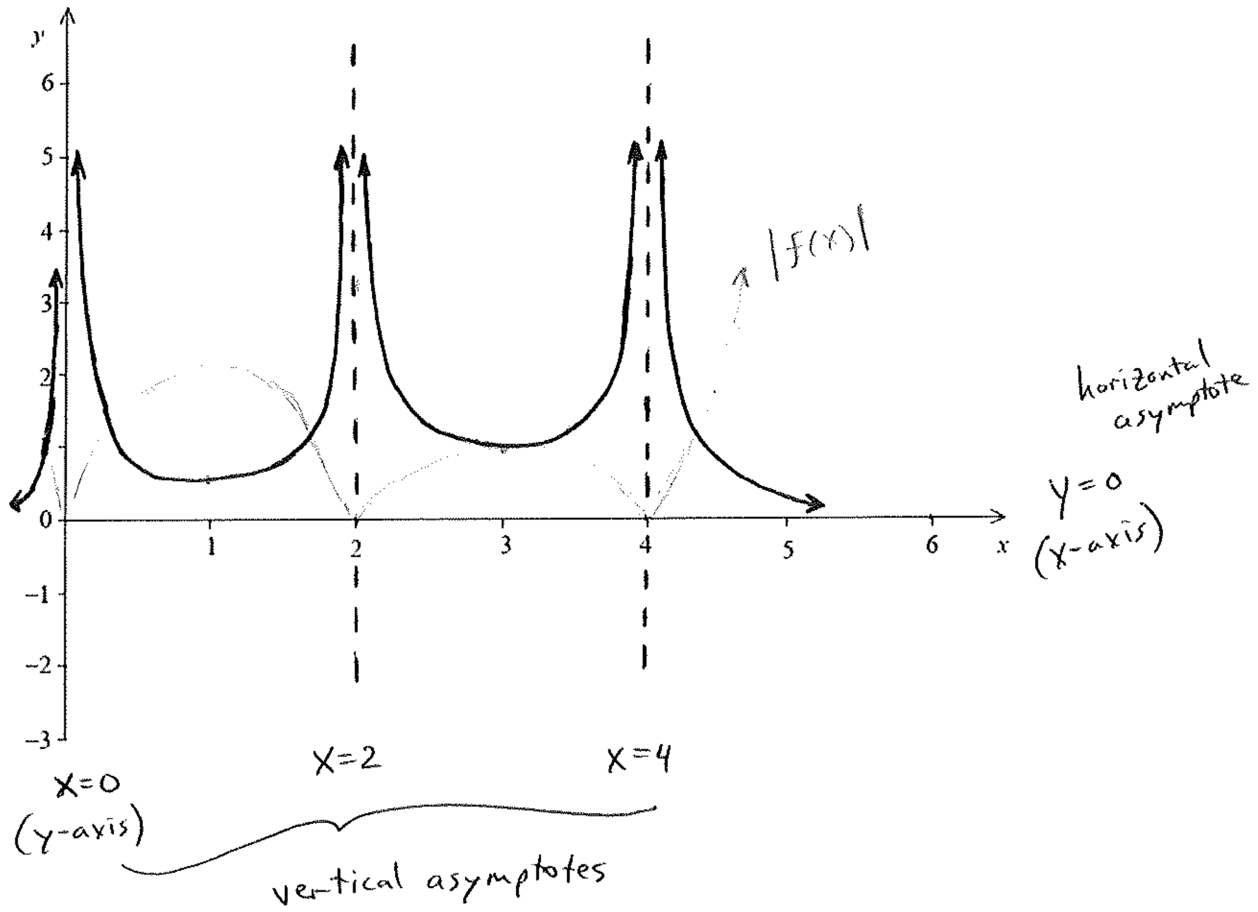
(d) $y = f\left(\dfrac{x}{2}\right)$



6. Below is the graph of the function $y = f(x)$ **[3 marks]**



On the coordinate plane below, sketch a graph $y = \dfrac{1}{|f(x)|}$



Part II - **calculator allowed – Qs** 7 - 9

7. Consider the functions $f(x)=\dfrac{2x+3}{x-4}$ and $g(x)=\dfrac{x-1}{x+1}$.

(a) State the domain and range for $f(x)$. **[2 marks]**

domain $x \in \mathbb{R},\ x \neq 4$ range $y \in \mathbb{R},\ y \neq 2$

(b) If $(c, 0)$ is the x-intercept for the graph of $f(x)$, then find the value of c. **[2 marks]**

$(-\frac{3}{2}, 0)$ is x-intercept $\rightarrow$ $c = -\frac{3}{2}$

(c) (i) Find $f^{-1}(x)$. $y = \dfrac{2x+3}{x-4}$ $x = \dfrac{2y+3}{y-4}$ **[3 marks]**

$x(y-4) = 2y+3 \rightarrow xy - 4x = 2y+3 \rightarrow xy - 2y = 4x+3$

$y(x-2) = 4x+3 \rightarrow y = \dfrac{4x+3}{x-2}$ $f^{-1}(x) = \dfrac{4x+3}{x-2}$

(ii) Why must $(0, c)$ be the y-intercept for the graph of $f^{-1}(x)$? **[2 marks]**

$(0, -\frac{3}{2})$ is the reflection of $(-\frac{3}{2}, 0)$ about the line $y=x$

and

(d) Find the value of $g(f(3))$. **[2 marks]**

$$g(f(3)) = g\left(\frac{2\cdot 3+3}{3-4}\right) = g(-9) = \frac{-9-1}{-9+1} = \frac{-10}{-8} = \frac{5}{4}$$

(e) Find an expression for $g(f(x))$. **[2 marks]**

$$g(f(x)) = \frac{\frac{2x+3}{x-4} - 1}{\frac{2x+3}{x-4} + 1} = \frac{\frac{2x+3}{x-4} - \frac{x-4}{x-4}}{\frac{2x+3}{x-4} + \frac{x-4}{x-4}} = \frac{\frac{2x+3-x+4}{x-4}}{\frac{2x+3+x-4}{x-4}} = \frac{x+7}{x-4} \cdot \frac{x-4}{3x-1}$$

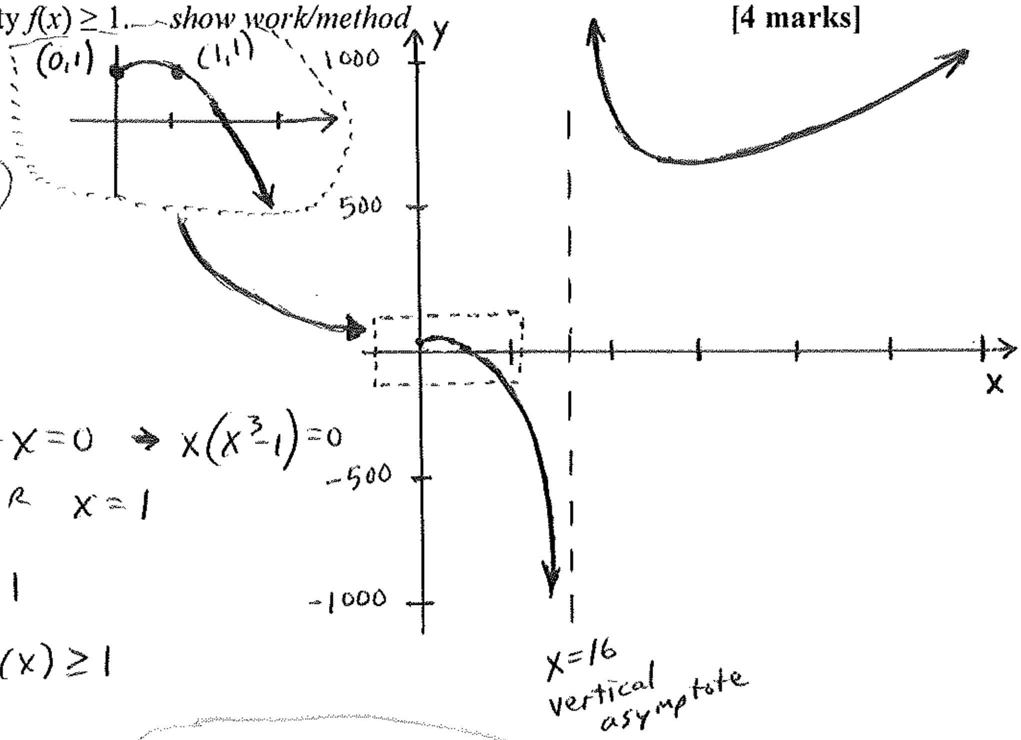
$$g(f(x)) = \frac{x+7}{3x-1}$$

over

8. Let $f(x) = \dfrac{4 - x^2}{4 - \sqrt{x}}$.

(a) State the largest possible domain for f. **[2 marks]**

(b) Solve the inequality $f(x) \geq 1$. – *show work/method* **[4 marks]**



(a)

domain $x \geq 0,\ x \neq 16$

(b) $4 - x^2 = 4 - \sqrt{x}$

$x^2 = \sqrt{x}$

$x^4 = x \Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$

$x = 0$ or $x = 1$

for $0 \leq x \leq 1$, $f(x) \geq 1$

also when $x > 16$, $f(x) \geq 1$

thus, solution for $f(x) \geq 1$ is $0 \leq x \leq 1$ or $x > 16$

9. State the domain and range for each function.

(a) $f(x) = \dfrac{1}{x^2 - 9}$ **[3 marks]**

domain $x \in \mathbb{R},\ x \neq \pm 3$

range $y \leq -\frac{1}{9}$ or $y > 0$

(b) $g(x) = \ln(x + 4)$ **[3 marks]**

domain $x > -4$

range $y \in \mathbb{R}$