Functions - Basics

WORKED SOLUTIONS

1. (a) domain: $\{x : -3 \le x \le 3\}$

range: $\{y : 0 \le y \le 3\}$

(b) domain: $x \in \mathbb{R}$

range: $y > 0$

(c) domain: $x \in \mathbb{R},\ x \neq \frac{\pm\sqrt{2}}{2}$

range: $y \le -1$ or $y > 0$

2.

(a) $h(x) = g(f(x)) = \dfrac{\frac{2}{x-4}}{2} - 1 = \dfrac{2}{x-4} \cdot \dfrac{1}{2} - 1 = \dfrac{1}{x-4} - 1 =$

$= \dfrac{1}{x-4} - \dfrac{x-4}{x-4} = \dfrac{-x+5}{x-4}$ thus, $h(x) = \dfrac{-x+5}{x-4}$

(b) $y = \dfrac{-x+5}{x-4} \rightarrow x = \dfrac{-y+5}{y-4} \rightarrow (y-4)x = -y+5 \rightarrow xy - 4x = -y+5$

$xy + y = 4x + 5 \rightarrow y(x+1) = 4x+5 \rightarrow y = \dfrac{4x+5}{x+1}$

thus, $h^{-1}(x) = \dfrac{4x+5}{x+1}$

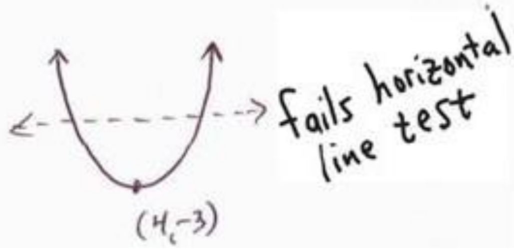
3. (a) $g(x) = 2(x^2 - 8x \quad\ ) + 29$

$= 2(x^2 - 8x + 16) + 29 - 32$

$g(x) = 2(x-4)^2 - 3$

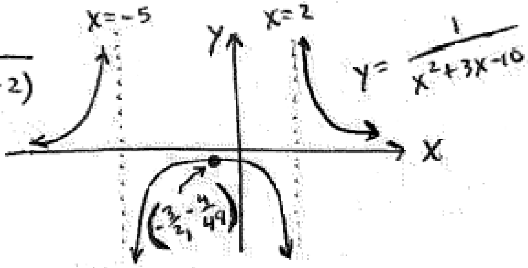
(b) vertex $(4, -3)$, axis of symmetry: $x = 4$

(c) No, function is many-to-one



4.

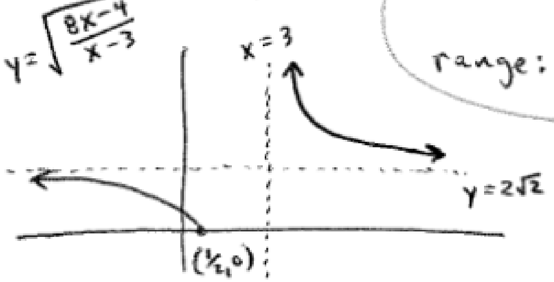
(a) $f(x)=\dfrac{1}{x^2+3x-10} = \dfrac{1}{(x+5)(x-2)}$



domain: $\{x : x \in \mathbb{R}, x \neq -5, x \neq 2\}$

range: $\{y : y \leq -\frac{4}{49}, y > 0\}$

(b) $g(x)=\sqrt{\dfrac{8x-4}{x-3}}$



domain: $\{x : x \leq -\frac{1}{2}, x > 3\}$

range: $\{y : y \in \mathbb{R}, y \neq 2\sqrt{2}\}$

5. $y = (x-3)^2 \rightarrow x = (y-3)^2 \rightarrow y - 3 = \pm\sqrt{x} \rightarrow y = 3 \pm \sqrt{x}$

but since range of $h^{-1}(x)$ is $y \geq 3$, then $y = 3 + \sqrt{x}$

thus, $h^{-1}(x) = 3 + \sqrt{x}$

domain: $\{x : x \geq 0\}$

range: $\{y : y \geq 3\}$

6.

(a) $h(g(x)) = \dfrac{1}{x^2+3-3} = \dfrac{1}{x^2}$

(b) $f(h(x)) = 2\left(\dfrac{1}{x+3}\right) - 1 = \dfrac{2}{x+3} - \dfrac{x+3}{x+3} = \dfrac{-x-1}{x+3}$

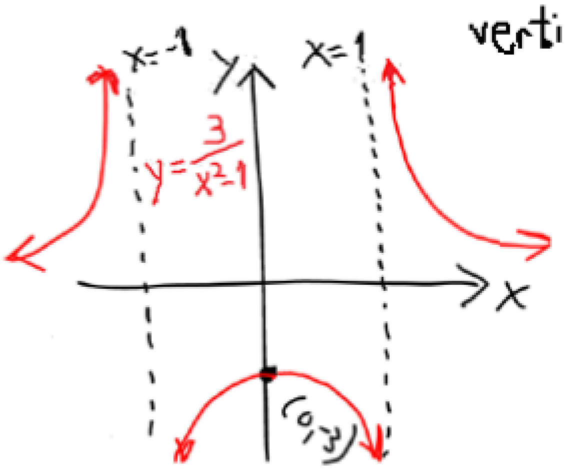
(c) $h^{-1}(x) = \dfrac{1-3x}{x}$; $g(h^{-1}(x)) = \left(\dfrac{1-3x}{x}\right)^2 - 3 =$

$= \dfrac{1-6x+9x^2}{x^2} - \dfrac{3x^2}{x^2} = \dfrac{1-6x+6x^2}{x^2}$

(d) $f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$; $f^{-1}(f(x)) = \frac{1}{2}(2x-1) + \frac{1}{2} =$

$= x - \frac{1}{2} + \frac{1}{2} = x$ Q.E.D.

7. (a)



vertical asymptotes:
$x = -1, \; x = 1$

horizontal asymptote:
$y = 0$ (x-axis)

(b) range: $\{ y \le -3 \text{ OR } y > 0 \}$