1

[Maximum mark: 7]

Consider the function $g(x) = 4\cos x + 1$, $a \le x \le \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c) For the value of a found in part (b),

(i) write down the domain of g^{-1};

(ii) find an expression for $g^{-1}(x)$. [3]

2

[Maximum mark: 5]

Find the value of the constant term in the expansion of $x^4\left(x + \frac{3}{x^2}\right)^5$.

3

[Maximum mark: 4]

Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$.

4

[Maximum mark: 5]

Consider the graphs of $y = |x|$ and $y = -|x| + b$, where $b \in \mathbb{Z}^+$.

(a) Sketch the graphs on the same set of axes. [2]

(b) Given that the graphs enclose a region of area 18 square units, find the value of b. [3]

5

[Maximum mark: 18]

Consider the polynomial $P(z) = z^5 - 10z^2 + 15z - 6$, $z \in \mathbb{C}$.

(a) Write down the sum and the product of the roots of $P(z) = 0$. [2]

(b) Show that $(z - 1)$ is a factor of $P(z)$. [2]

The polynomial can be written in the form $P(z) = (z - 1)^3\left(z^2 + bz + c\right)$.

(c) Find the value of b and the value of c. [5]

(d) Hence find the complex roots of $P(z) = 0$. [3]

Consider the function $q(x) = x^5 - 10x^2 + 15x - 6$, $x \in \mathbb{R}$.

(e) (i) Show that the graph of $y = q(x)$ is concave up for $x > 1$.

(ii) Sketch the graph of $y = q(x)$ showing clearly any intercepts with the axes. [6]