1

[Maximum mark: 6]

The function f is defined by $f(x) = \sqrt[3]{2x+1}$, $-14 \le x \le 13$.

(a) Write down the range of f.   [2]

(b) Find an expression for f^{-1}.   [2]

(c) Write down the domain and range of f^{-1}.   [2]

2

[Maximum mark: 5]

The quadratic equation $x^2 - kx + (k-1) = 0$ has roots α and β. Without solving the equation, find the possible values of the real number k given that $\alpha^2 + \beta^2 = 17$.

3

[Maximum mark: 6]

Given that $(x-4)$ is a factor of $f(x) = x^3 - 2x^2 + ax + b$ and that division of $f(x)$ by $(x+2)$ leaves a remainder of 18, find the value of a and the value of b.

4

[Maximum mark: 8]

Let $p(x) = \dfrac{1}{4}x^5 - 2x^4 - 5x^3 + 40x^2 + 16x - 128$, $x \in \mathbb{R}$.

(a) For the polynomial equation $p(x) = 0$, state

  (i)   the sum of the roots;

  (ii) the product of the roots.   [3]

A new polynomial is defined by $q(x) = p(2x - 2)$. For the polynomial equation $q(x) = 0$, find

(b) (i)   the sum of the roots;

  (ii) the product of the roots.   [5]

5

[Maximum mark: 17]

The function f is defined by $f(x) = 1 + \dfrac{4x}{x+3}$, $x \in \mathbb{R}$, $x \ne -3$.

(a) Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes.   [4]

(b) Find an expression for f^{-1}.   [4]

(c) Find all values of x for which $f(x) = f^{-1}(x)$.   [3]

(d) Solve the inequality $|f(x)| < 2$.   [4]

(e) Solve the inequality $f(|x|) < 2$.   [2]