1

we get

$$\begin{aligned}(2x-3)^4 &= (2x)^4 + 4\cdot(2x)^3\cdot(-3) + 6\cdot(2x)^2\cdot(-3)^2 + 4\cdot(2x)\cdot(-3)^3 + (-3)^4 \\ &= 16x^4 - 96x^3 + 216x^2 - 216x + 81.\end{aligned}$$

2

Solution

The third term in the expansion of $(x+k)^8$ is given by

$$\begin{aligned}\binom{8}{6}(x)^6(k)^2 &= 252x^6 \\ (28)(\cancel{x^6})(k^2) &= 252\cancel{x^6} \\ k^2 &= \frac{252}{28} \\ k^2 &= 9 \qquad \text{[by using G.D.C.]} \\ k &= \pm 3.\end{aligned}$$

3

Solution

We have

$$\begin{aligned}\binom{6}{2}\left(\frac{x^2}{2}\right)^2\left(\frac{a}{x}\right)^4 &= 960 \\ 15\left(\frac{\cancel{x^4}}{4}\right)\left(\frac{a^4}{\cancel{x^4}}\right) &= 960 \\ \frac{a^4}{4} &= 64 \qquad \text{[since } 960/15 = 64\text{]} \\ a^4 &= 256 \\ a &= \pm\sqrt[4]{256} \\ a &= \pm 4. \qquad \text{[by using G.D.C.]}\end{aligned}$$

4

Solution

(a) $3x^2 + 5x - 2 = (3x-1)(x+2)$.

(b) By binomial expansion formula, we have

$$
\begin{aligned}
(3x^2+5x-2)^5 &= (3x-1)^5(x+2)^5 \\
&= \left(\sum_{k=0}^{5}\binom{5}{k}\cdot(3x)^{5-k}\cdot(-1)^k\right)\left(\sum_{k=0}^{5}\binom{5}{k}\cdot x^{5-k}\cdot 2^k\right) \\
&= \left(\sum_{k=0}^{5}\binom{5}{k}\cdot 3^{5-k}\cdot(-1)^k\cdot x^{5-k}\right)\left(\sum_{k=0}^{5}\binom{5}{k}\cdot 2^k\cdot x^{5-k}\right)
\end{aligned}
$$

Hence the coefficient of x^9 in the expansion of $(3x^2+5x-2)^5$ is

$$
\begin{aligned}
\left[\binom{5}{0}\cdot 3^{5-0}\cdot(-1)^0\right]\cdot\left[\binom{5}{1}\cdot 2^1\right]+\left[\binom{5}{1}\cdot 3^{5-1}\cdot(-1)^1\right]\cdot\left[\binom{5}{0}\cdot 2^0\right] &= [3^5]\cdot[10]-[5\cdot 3^4]\cdot[1] \\
&= 2025. \quad \text{[by using G.D.C.]}
\end{aligned}
$$

5

Solution

We have

$$
\begin{aligned}
x\binom{7}{2}(2x^2)^2\left(\frac{k}{x}\right)^5 &= 20,412 \\
\cancel{x}(21)(4\cancel{x^4})\left(\frac{k^5}{\cancel{x^5}}\right) &= 20,412 \\
84k^5 &= 20,412 \\
k^5 &= 243 \qquad \text{[by using G.D.C.]} \\
k &= \sqrt[5]{243} \\
k &= 3. \qquad \text{[by using G.D.C.]}
\end{aligned}
$$

6

Solution

(a) The term in x^2 in the expansion of $(2x+1)^5$ is given by

$$\begin{aligned}\binom{5}{2}(2x)^2(1)^3 &= 10(4x^2)(1)\\ &= 40x^2.\end{aligned}$$

(b) Hence the term in x^3 in the expansion of $(x+3)(2x+1)^5 = x(2x+1)^5 + 3(2x+1)^5$ is given by

$$\begin{aligned}x\cdot(x^2\text{ term}) + 3\cdot(x^3\text{ term}) &= x(40x^2) + 3\binom{5}{3}(2x)^3(1)^2 \qquad \text{[by part (a)]}\\ &= 40x^3 + 3(10)(8x^3)(1)\\ &= 40x^3 + 240x^3\\ &= 280x^3.\end{aligned}$$

7

Solution

(a) By binomial expansion formula, we have

$$\begin{aligned}(3-y)^5 &= ((-y)+3)^5\\ &= \sum_{k=0}^{5}\binom{5}{k}\cdot(-y)^{5-k}\cdot 3^k\\ &= -y^5 + 15y^4 - 90y^3 + 270y^2 - 405y + 243\end{aligned}$$

in descending order of powers as required.

(b) If we substitute $y = 0.1$ into the binomial expansion given above, then we get

$$\begin{aligned}(2.9)^5 &= (3-0.1)^5\\ &= -(0.1)^5 + 15(0.1)^4 - 90(0.1)^3 + 270(0.1)^2 - 405(0.1) + 243\\ &= -0.00001 + 0.0015 - 0.09 + 2.7 - 40.5 + 243\\ &= -40.59001 + 245.7015\\ &= 205.11149\end{aligned}$$

8

Solution

The term in x^2 in the expansion of $(2x+1)^n$ is given by

$$\begin{aligned}
\binom{n}{2}(2x)^2(1)^{n-2} &= 40nx^2 \\
\frac{n!}{2!(n-2)!}\cdot(4\cancel{x^2})(1) &= 40n\cancel{x^2} \\
\frac{n(n-1)}{2}\cdot 4 &= 40n \\
n(n-1) &= 20n \\
n^2-21n &= 0 \\
n(n-21) &= 0 \\
n &= 21.
\end{aligned}$$

Solution

The term in x^3 in the expansion of $x(2x+1)^n$ is given by

$$\begin{aligned}
x\binom{n}{2}(2x)^2(1)^{n-2} &= 20nx^3 \\
\cancel{x}\cdot\frac{n!}{2!(n-2)!}\cdot(4\cancel{x^2})(1) &= 20n\cancel{x^3} \\
\frac{n(n-1)}{2}\cdot 4 &= 20n \\
n(n-1) &= 10n \\
n^2-11n &= 0 \\
n(n-11) &= 0 \\
n &= 11.
\end{aligned}$$

Solution

If we expand $\left(\frac{1}{2x}+3x\right)^5$, then we get

$$\begin{aligned}\left(\frac{1}{2x}+3x\right)^5 &= \sum_{k=0}^{5}\binom{5}{k}\cdot\left(\frac{1}{2x}\right)^{5-k}\cdot(3x)^k\\ &= \sum_{k=0}^{5}\binom{5}{k}\cdot\left(\frac{3^k}{2^{5-k}}\right)\cdot x^{2k-5}\\ &= \frac{1}{32}x^{-5}+\frac{15}{16}x^{-3}+\frac{45}{4}x^{-1}+\frac{135}{2}x+\frac{405}{2}x^3+243x^5.\end{aligned}$$

If we expand $(x+1)^4$, then we get

$$\begin{aligned}(x+1)^4 &= \sum_{k=0}^{4}\binom{4}{k}\cdot x^{4-k}\\ &= x^4+4x^3+6x^2+4x+1.\end{aligned}$$

Then the coefficient of $\frac{1}{x}$ in the product $\left(\frac{1}{2x}+3x\right)^5(x+1)^4$ is

$$\begin{aligned}\frac{1}{32}+\frac{15}{16}\cdot 6+\frac{45}{4} &= \frac{1+180+360}{32}\\ &= \frac{541}{32}\\ &= 16.90625. \qquad \text{[by using G.D.C.]}\end{aligned}$$

Solution

We have

$$(1+x)^3(1+px)^4 = (1+3x+3x^2+x^3)(1+4px+6p^2x^2+4p^3x^3+p^4x^4).$$

Hence we get

$$\begin{aligned} x \text{ terms} &: \quad 4px+3x = qx \implies 4p+3 = q, \\ x^2 \text{ terms} &: \quad 6p^2x^2+12px^2+3x^2 = 93x^2 \implies 6p^2+12p+3 = 93. \end{aligned}$$

Solving the last equation for p, we get

$$\begin{aligned} 6p^2+12p+3 &= 93 \\ 6p^2+12p-90 &= 0 \\ p^2+2p-15 &= 0 \\ (p+5)(p-3) &= 0 \\ p &= -5, 3. \end{aligned}$$

Hence we get

$$p=-5,\ q=-17$$

$$\text{and}$$

$$p=3,\ q=15.$$