Vector calculus

Monday, August 17, 2020   11:00 AM

vector fields:

vector field is a function whose domain is a set of points in R^2 (or R-3) and whose range is set of vectors.

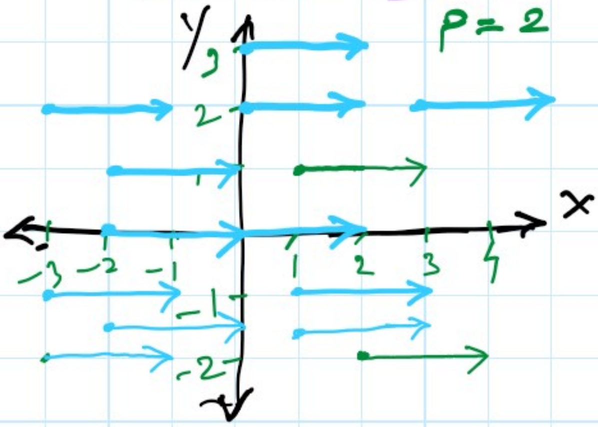
In R-2 ; $F(x,y) = P\,\hat{i} + Q\,\hat{j}$

In R-3 ; $F(x,y,z) = P\,\hat{i} + Q\,\hat{j} + R\,\hat{k}$

P, Q, R are defined function.

Ex: $F(x,y) = 2\,\hat{i}$, $(x,y) \to$ R-2

$P = 2$



1) plug in (x,y)
2) This gives a vector
3) plot the vector at a point.

$(1,1) \Rightarrow F(1,1) = 2\,\hat{i}$

Ex $F(x,y) = -x\,\hat{i} - y\,\hat{j}$

$(x,y) \to$ R-2

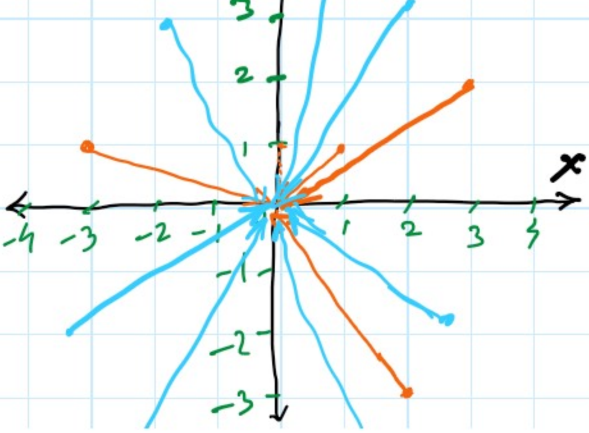
$(1,1) \to F(1,1) = -\hat{i} - \hat{j}$

$(3,2) \to F(3,2) = -3\hat{i} - 2\hat{j}$

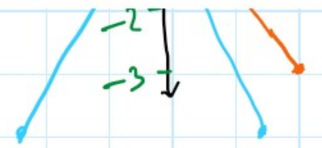
$(-3,1) \to F(-3,1) = 3\hat{i} - \hat{j}$

$(2,-3) \to F(2,-3) = -2i + 3j$

$(-2,-1) \to$



$(-2,-1) \rightarrow$



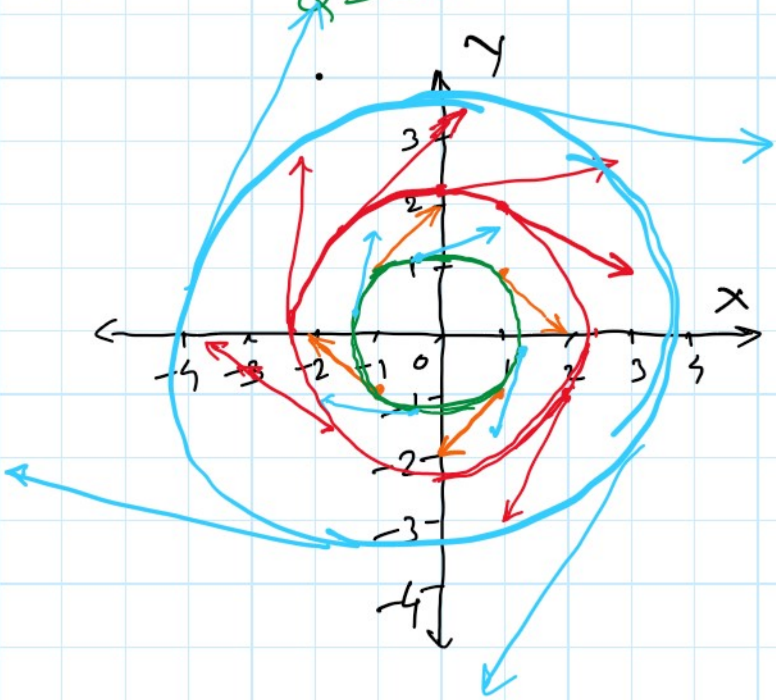
All vectors directed towards origin.

$\underline{\underline{Ex:}}$ $F(x,y) = y\,\hat{i} - x\,\hat{j}$

$P = y$
$Q = -x.$

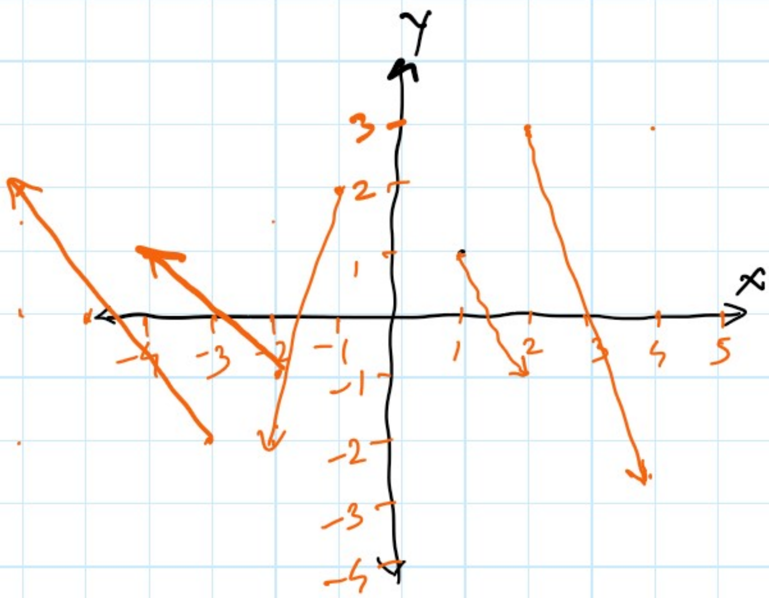
$(x,y) \rightarrow R\text{-}2.$

$(1,1) \rightarrow \hat{i} - \hat{j}$
$(1,-1) \rightarrow -\hat{i} - \hat{j}$
$(-1,-1) \rightarrow -\hat{i} + \hat{j}$
$(-1,1) \rightarrow \hat{i} + \hat{j}$
$(1,2) \rightarrow 2\hat{i} - \hat{j}$
$(2,-1) \rightarrow -\hat{i} - 2\hat{j}$
$(0,-2)$



$\underline{\underline{Ex}}$ $F(x,y) = x\,\hat{i} - 2y\,\hat{j}$

$(1,1) \rightarrow i - 2j$
$(2,3) \rightarrow 2i - 6j$
$(-1,2) \rightarrow -i - 4j$
$(-2,-1) \rightarrow -2i + 2j$
$(-3,-2) \rightarrow -3i + 4j$



Ex $F(x,y) = \left(\frac{x}{\sqrt{x^2+y^2}}\right)\hat{i} + \left(\frac{y}{\sqrt{x^2+y^2}}\right)\hat{j}$

$$\sqrt{\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}} = \sqrt{1} = 1.$$

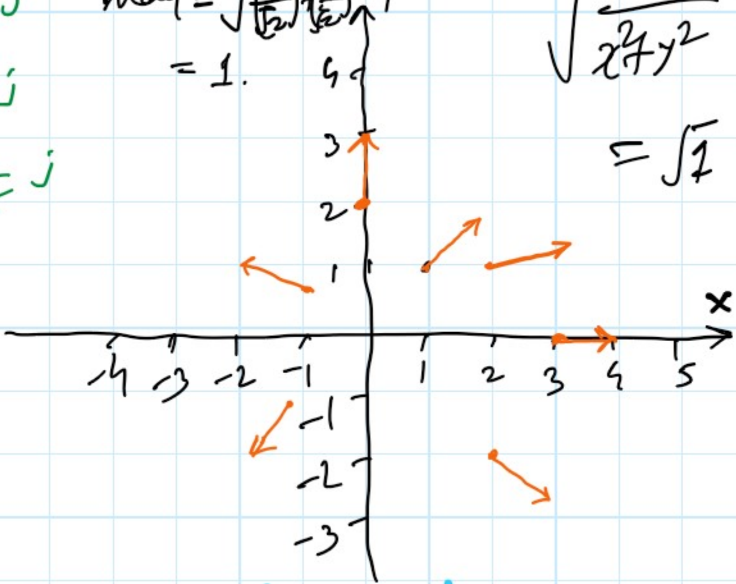
$(1,1) \rightarrow \left(\frac{1}{\sqrt{2}}\right) i + \left(\frac{1}{\sqrt{2}}\right) j$ mag $= \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1.$

$(1,-1) \rightarrow \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$

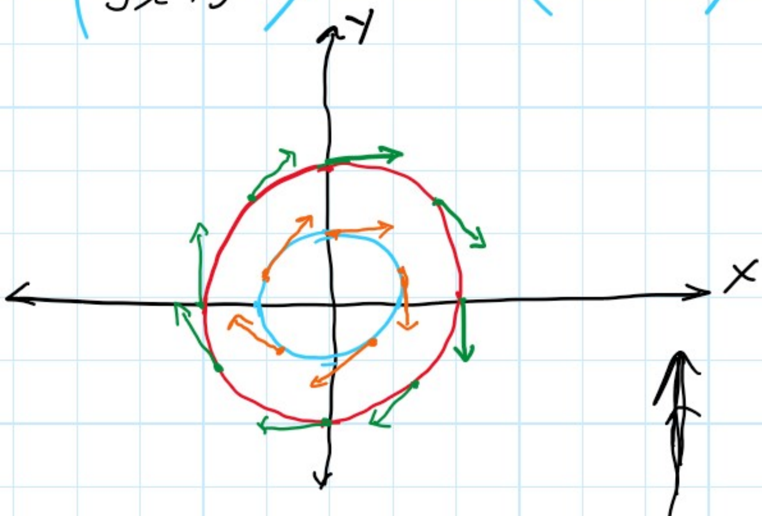
$(2,1) \rightarrow \frac{2}{\sqrt{5}} i + \frac{1}{\sqrt{5}} j$

$(0,2) \rightarrow j$

$(3,0) \rightarrow i$



Ex $F(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}\right)\hat{i} - \left(\frac{x}{\sqrt{x^2+y^2}}\right)\hat{j}$

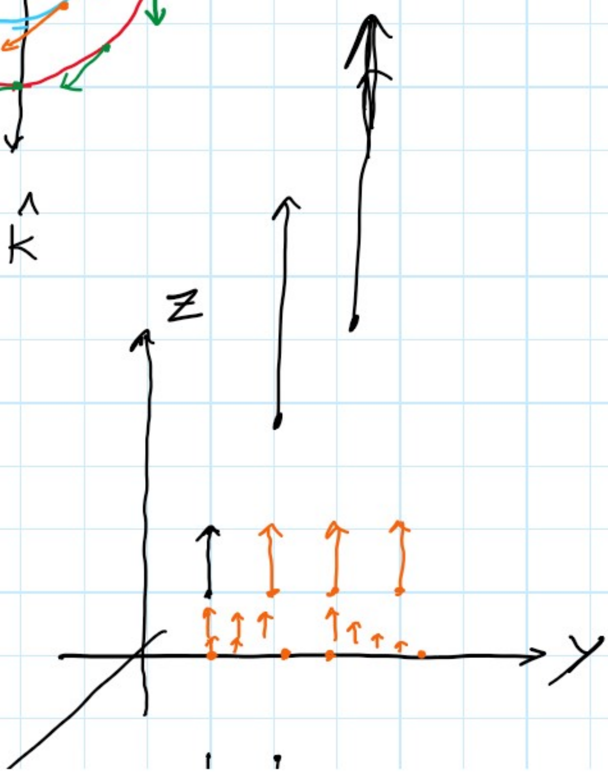


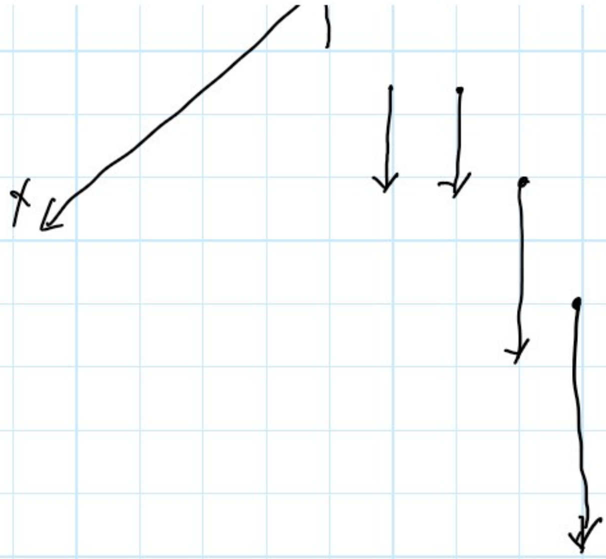
Ex $F(x,y,z) = z\,\hat{k}$

R^{3}

$(1,1,1) \Rightarrow \hat{k}$

$(1,1,-2) \Rightarrow -2\hat{k}$





$\nabla f(x,y)$ ← Gradient
or $\nabla f(x,y,z)$

Ex $f(x,y) = x^2y - y^3$

$\nabla f(x,y) = f_x\,\hat{i} + f_y\,\hat{j}$

$\nabla f(x,y) = (2xy)\,\hat{i} + (x^2 - 3y^2)\,\hat{j}$

$F(x,y) = P\,\hat{i} + Q\,\hat{j}$ vector field.

→ conservative vector field. because it is gradient of some potential f^n.

If the given vector field is gradient of some potential function then the given vector field is said to be conservative vector field.

Ex $F(x,y,z) = y \ln(x+z)$

Ex $f(x,y,z) = y \ln(x+z)$

find the conservative vector field.

$$F(x,y,z) = \nabla f(x,y,z) = \overbrace{\left(\frac{y}{x+z}\right)}^{f_x}\hat{i} + \overbrace{\ln(x+z)}^{f_y}\hat{j} + \overbrace{\left(\frac{y}{x+z}\right)}^{f_z}\hat{k}$$