Test (spherical coordinate sys, change of variable)

Friday, August 14, 2020 4:07 PM

Each Question carries 5 Marks.

1) $\iint_R (4x+8y)\,dA$, where R is is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$ & $(1,5)$;

$x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$. $\int_{-4}^{4}\int_{0}^{8} (3v-5u)\frac{1}{4}\,dv\,du$

ANS: -192

SCORE:- M -0 -0
D -112 -0
S -192 -5

2) $\iint_R (x+y)e^{x^2-y^2}\,dA$, where R is the rectangle enclosed by the lines $x-y=0$, $x-y=2$, $x+y=0$, $x+y=3$.

ANS:- $\frac{1}{4}(e^6-7)$ $\int_0^3\int_0^2 u\,e^{uv}\left(-\frac{1}{2}\right)dv\,du$

Cumulative Marks: (0) M —
(0) D $\frac{1}{4}(e^6+7)$
(10) S $\frac{1}{4}(e^6-7)$

3) $\iint_R e^{x+y}\,dA$, where R is given by the inequality $|x|+|y| \le 1$.

ANS: $e-\frac{1}{e}$

C.M. D $e-1/e$ 5
M — 0
S $e-2$ 10

4) Evaluate the integral by changing to spherical co-ordinates.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx.$$

ANS:- $\frac{4\sqrt{2}-5}{15}$

C.M:

S	$\frac{4\sqrt{2}-5}{15}$ ✓	15
D	$\frac{2^{5/2}-7}{15}$	5
M	—	0

5) Use spherical coordinates.

Evaluate $\iiint_E x e^{x^2+y^2+z^2} \, dV$, where E is the portion of the unit ball $x^2+y^2+z^2 \leq 1$ that lies in the first octant.

ANS. $\pi/8$

C.M.

D	$\pi/8$	10
S	$\pi/8$	20
M	$\pi/4$	0

6) Use spherical coordinates.

Evaluate $\iiint_E xyz \, dV$, where E lies between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \pi/3$.

ANS:- 0

C.M.

M	189	
S	—	
D	189	

Manav (0) Dev (10) Shawn (20)

Manav

$\begin{pmatrix} 0 \\ 30 \end{pmatrix}$

$\begin{pmatrix} 10 \\ 30 \end{pmatrix}$

$\begin{pmatrix} 20 \\ 30 \end{pmatrix}$