Test paper Revisit

Wednesday, August 5, 2020 2:53 PM

2) Ans: $\left(0, \frac{45}{14\pi}\right)$

$$M = \int_0^{\pi}\int_1^{2} kr^2 \, dr\, d\theta = \frac{7}{3}\pi K$$

The region is symmetry about y-axis. The com has to be on y axis.

$$M_y = \int_0^{\pi}\int_1^{2} (r\cos\theta) kr \, r\, dr\, d\theta = 0.$$

$$M_x = \int_0^{\pi}\int_1^{2} r\sin\theta \; kr \, r\, dr\, d\theta = \frac{15}{2}K.$$

$$COM \equiv (\bar{x}, \bar{y}) = \left(0, \frac{45}{14\pi}\right)$$

③ $2x+y+z=4$

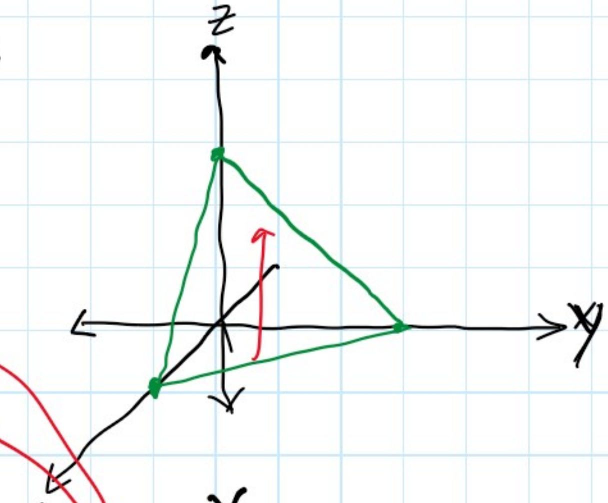
R-3 z-simple:

$z=0$,

$z = 4-2x-y$

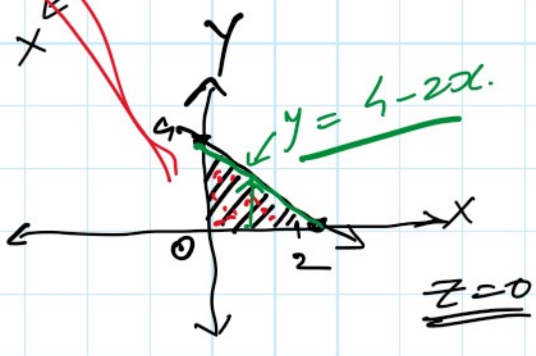
$z=4$

$\boxed{0 \le z \le 4-2x-y}$



Draw Region R: - XY

$z=0$



$0 = 4-2x-y$

$2x+y=4$

$\int_{}^{2}\int^{y=4-2x}\int^{z=4-2x-y}\left(\quad \right) dz\, dy\, dx.$

$$V = \int_{x=0}^{2} \int_{y=0}^{y=4} \int_{z=0}^{z=\ldots} (1) \, dz \, dy \, dx$$

$$= \frac{16}{3}$$

(4) $y = x^2 + z^2$ & $y = 8 - x^2 - z^2$

$\iiint$

$y = f(x, z)$

R-3 y – Simple:-

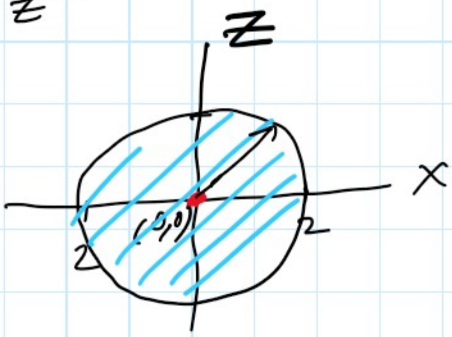
$$x^2 + z^2 \le y \le 8 - x^2 - z^2$$

R-2 Draw region R: (xz-plane). (level curve).

$$x^2 + z^2 = 8 - x^2 - z^2$$

$$\Rightarrow 2x^2 + 2z^2 = 8$$

$$\Rightarrow x^2 + z^2 = 4$$



$$0 \le r \le 2$$

$$0 \le \theta \le 2\pi$$

$$V = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} \int_{y=x^2+z^2}^{y=8-x^2-z^2} 1 \cdot r \, dy \, dr \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{2} \int_{y=r^2}^{y=8-r^2} 1 \cdot r \, dy \, dr \, d\theta$$

$$= \int_{\theta=0} \int_{r=0} \int_{y=r^2}$$

$$= 16\pi$$

5) $\dfrac{8\pi}{3} + \dfrac{128}{15}$

$$\iiint_E (x+y+z)\, dV$$

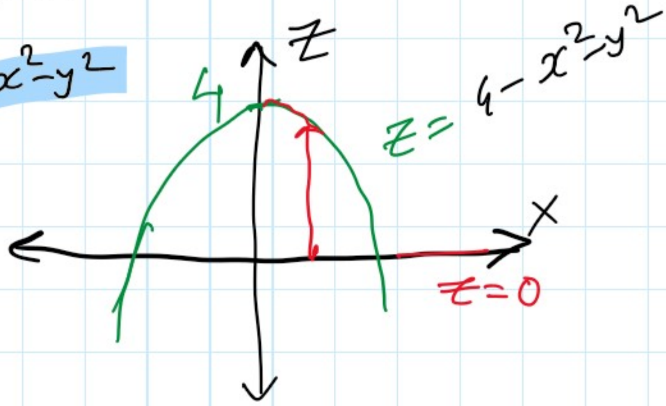
Z-simple:

Sol^n $\longrightarrow$ $z = 4 - x^2 - y^2$

$z = 0$ — Ist octant.

$0 \le z \le 4 - x^2 - y^2$

$z = f(x,y)$



$z = 4 - x^2 - y^2$

$\xrightarrow[\text{XZ-plane}]{Y=0}$

$z = 4 - x^2$

R-2 (Draw region in xy plane)

$z = 4 - x^2 - y^2$

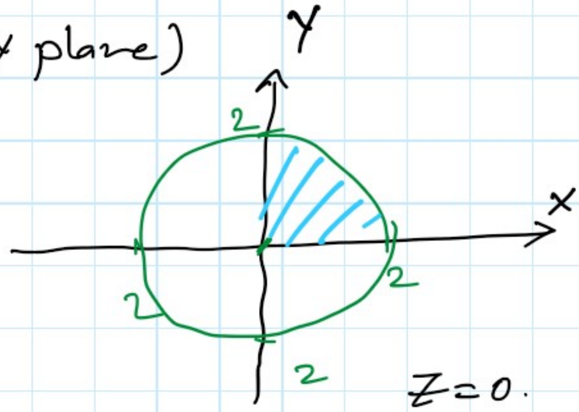
on xy-plane $\big\downarrow$ $z = 0$

$0 = 4 - x^2 - y^2$

$x^2 + y^2 = 4$

$0 \le r \le 2$

$0 \le \theta \le \pi/2$



$z = 0.$

$\theta = \pi/2 \quad r = 2 \quad z = 4 - x^2 - y^2$

$$V = \int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=2}\int_{z=0}^{z=4-x^2-y^2} x+y+z \; r \cdot dz \, dr \, d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=2}\int_{z=0}^{z=4-r^2} (r\cos\theta + r\sin\theta + z)\, r \; dz \, dr \, d\theta.$$

6 $\iiint_E x \, dV$

$z=0, \quad z = x+y+5$

$x^2+y^2=4 \quad \& \quad x^2+y^2=9$

R-3 – z-simple:

$$0 \le z \le x+y+5$$

Draw region R: (xy-) plane.

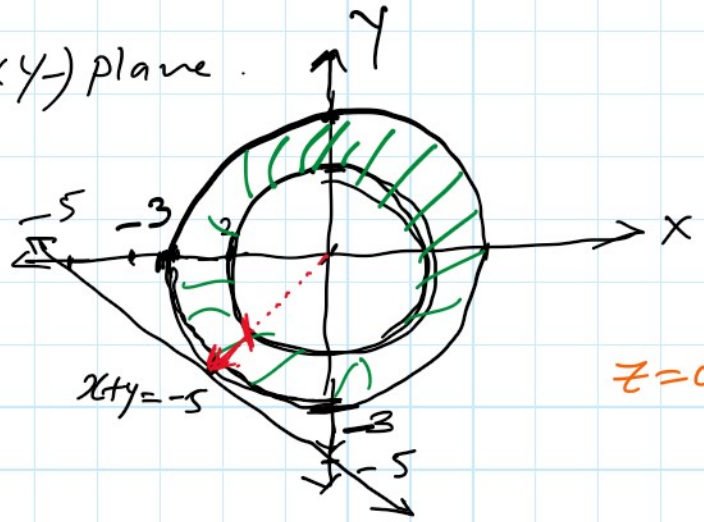
$z = x+y+5$

$\downarrow z=0$

$x+y=-5$

$x^2+y^2=4,$

$x^2+y^2=9.$



$z=0$

$2 \le r \le 3$

$0 \le \theta \le 2\pi$

$z = r\cos\theta + r\sin\theta + 5$

$0 \le \theta \le 2\pi$

$z = r\cos\theta + r s\ldots$

$$I = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{2}\int_{z=0}^{z=x+y+5} (\, r\cos\theta \,)\, r\, dz\, dr\, d\theta$$

$$= \frac{65\pi}{4}$$

7) $\rho(x, y, z) = K.$

$x = y^2$, $x = z$, $z = 0$ & $x = 1$.

Sol^n:

$z = 0$ & $z = x$.

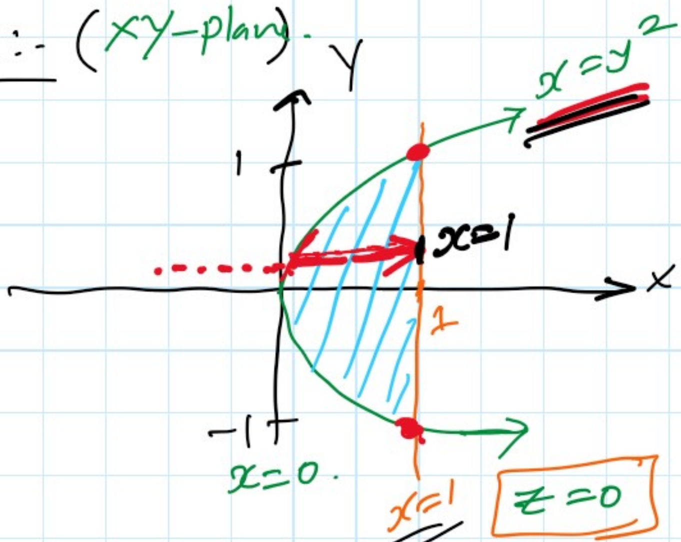
① R-3 z-simple

$0 \le z \le x$

(2) Draw the region R:- (xy-plan).

$x = y^2$

$x = z \xrightarrow[xy\,pl_{n}]{z=0} x = 0.$



$1 = y^2$

$y = \pm 1$

$$m = \int_{y=-1}^{y=1}\int_{x=y^2}^{x=1}\int_{z=0}^{z=x} (\; K. \;)\, dz\, dx\, dy$$

$$= K \int_{-1}^{1} \int_{x=y^2}^{x=1} [z]_0^{x} \, dx \, dy$$

$$= K \int_{-1}^{1} \int_{x=y^2}^{x=1} x \, dx \, dy$$

$$= K \int_{-1}^{1} \left[\frac{x^2}{2} \right]_{y^2}^{1} dy$$

$$= \frac{K}{2} \int_{-1}^{1} 1 - y^4 \, dy =$$

$$= K \int_{0}^{1} (1 - y^4) \, dy = K \left[y - \frac{y^5}{5} \right]_0^1$$

$$= K \left(1 - \frac{1}{5} \right) = \frac{4K}{5}$$

Because of the symmetry of E and ρ about the xz-plane . .

$M_{xz} = 0$ & $\therefore$ $\bar{y} = 0$.

$$M_{yz} = \iiint_E x K \, dz \, dx \, dy$$

$$M_{xy} = \iiint_E z K \, dz \, dx \, dy.$$

$COM(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{5}{7}, 0, \frac{5}{14}\right)$