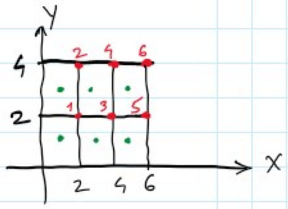
Double Integral Revision Practice (Double Integration).

Monday, August 3, 2020 10:55 AM

1) a).

$f(x,y) = xy$



$$V = \sum_{i=1}^{3}\sum_{j=1}^{2} f(x_i, y_j)\,\Delta A.$$

$$= [f(2,2) + f(2,4) + f(4,2) + f(4,4) + f(6,2) + f(6,4)] \times 4$$

$$= [4 + 8 + 8 + 16 + 12 + 24]\,4 = 288$$

1-b $$V = \sum_{i=1}^{3}\sum_{j=1}^{2} f(x_i, y_j)\,\Delta A$$

$f(1,1)$ $f(1,3)$ $f(3,1)$ $f(5,3)$

$$= [1 + 3 + 3 + 9 + 5 + 15] \times 4$$

$f(3,3)$ $f(5,1)$

$$= 144$$

2) -12, -8

3) 0

4) $\pi/3$

5) 2

6) $Z = 16 - x^2$, $y = 5$

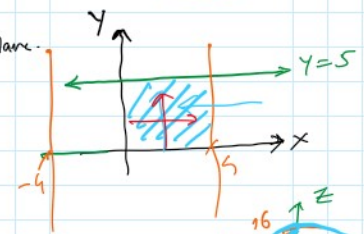
$Z = 0$ on XY plane.

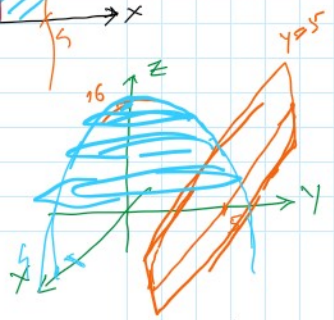
$16 - x^2 = 0.$

$x^2 = 16$

$x = 4$

$x = \pm 4$





$$V = \int_{y=0}^{5}\int_{x=0}^{4} (16 - x^2)\,dx\,dy$$

7) $$V = \int_{0}^{4}\int_{-1}^{1} [2 + x^2 + (y-2)^2]\,dx\,dy - \int_{0}^{4}\int_{-1}^{1} (1)\,dx\,dy$$

Q $Z = 1$ & $Z = 2 + x^2 + (y-2)^2$

I^{st} octant.

To find out the region on XY plane. when two surfaces are given,
we need to find out level curve by setting them equal.

$1 = 2 + x^2 + (y-2)^2$

$x^2 + y^2 = r^2$

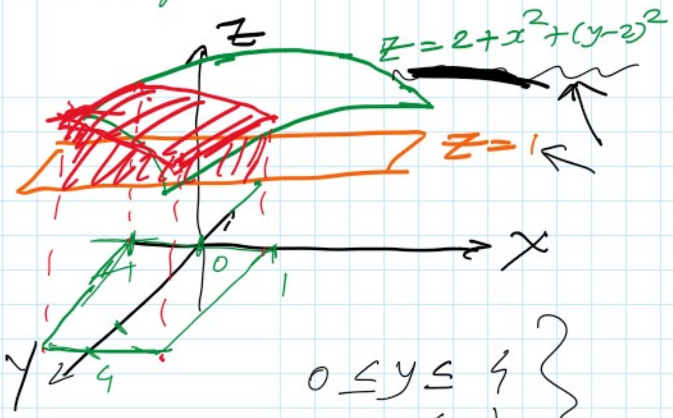
$y = r\sin\theta$

by setting them equal.

$$1 = 2 + x^2 + (y-2)^2$$

take projection of this on XY plane

Required Region



$0 \le y \le 4$

$-1 \le x \le 1$

$$1 = 2 + x^2 + (y-2)^2$$

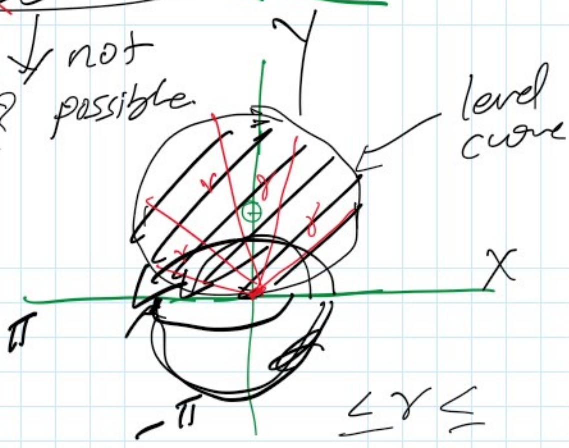
$$x^2 + (y-2)^2 = 4$$

circle.

$y = r\sin\theta$

not possible

$r = ?$



$\le r \le$

$0 \le \theta \le \pi$

8) Ans: $40 - 8\ln 5$

$$V = \int_0^2 \int_0^4 (x+2y) - \frac{2xy}{x^2+1}\, dy\, dx.$$

9) $\iint xy\, dA$

10-a $$= \int_{-1}^{2} \int_{y^2}^{y+2} y\, dx\, dy = \left(\frac{9}{4}\right)$$

10-b $$\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy = \left(\frac{1}{2}e^{16} - \frac{17}{2}\right)$$

$$y^{\cancel{2}} \frac{e^{xy}}{\cancel{y}} = \int_0^4 y e^{xy}\, dy$$

by parts.

11) $\frac{16\pi}{3}$

12) 81π

1) Find the mass & center of mass of lamina that occupies the region D and has given density function ρ.

D is bounded by $y = x^2$ & $y = x+2$;

$\rho(x,y) = kx$.

Soln: draw the region.

Sol: draw the region.

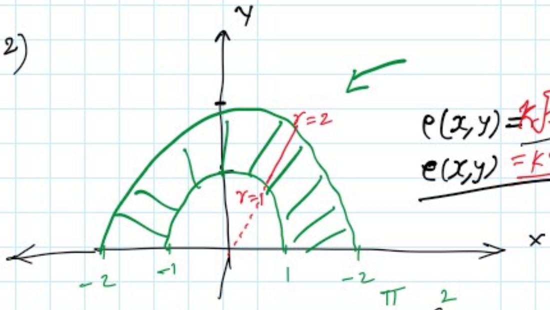
$$M = \int_{-1}^{2}\int_{x^2}^{x+2} kx \, dy \, dx = \frac{9}{4}k.$$

$$M_y = \int_{-1}^{2}\int_{x^2}^{x+2} kx^2 \, dy \, dx = \frac{63}{20}k.$$

$$M_x = \int_{-1}^{2}\int_{x^2}^{x+2} kxy \, dy \, dx = \frac{45}{8}k.$$

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right) = \left(\frac{7}{5}, \frac{5}{2}\right)$$

2)



$\rho(x,y) = \cancel{k\sqrt{x^2+y^2}}$

$\rho(x,y) = kr$

$$M = \int_{\theta=0}^{\pi}\int_{r=1}^{2} kr \, r \, dr \, d\theta$$

✓ $M_x =$

✓ $M_y = 0$