Probability Distribution

Thursday, April 15, 2021 5:32 AM

Binomial distribution.

Random variable: (discrete random variable / continuous random variable)

Normal distribution.

Random Variable →

probability density f^n →

X	1	2	3	4	5	6
p(x)	1/6	1/6	1/6	1/6	1/6	1/6

← probability distribution table.

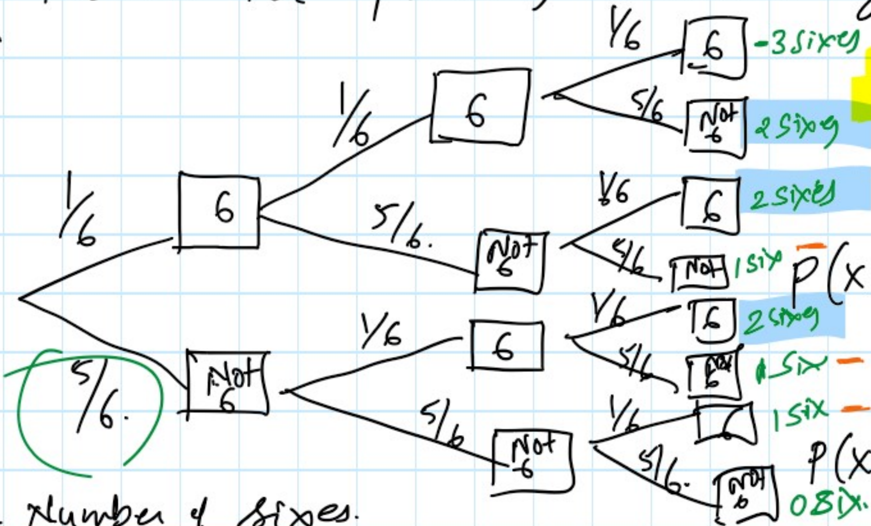
$$\sum p(x) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1.$$

$$E(X) = \sum x \cdot p(x) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = 3.5$$

1) Let x be the random variable that represent the number of sixes obtained when a dice is rolled three times.

Tabulate the probability distribution for x.

Solⁿ:



X - 0, 1, 2, 3

$$P(X=0) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$$P(X=1) = \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{75}{216} = \frac{25}{72}$$

x - Number of sixes.

X	0	1	2	3
p(x)	125/216	25/72	5/72	1/216

$$P(X=2) = \frac{15}{216} = \frac{5}{72}$$

$$P(X=3) = 1/216$$

$\sum p(x) = 1.$ pdf

x	1	2	3	4
p(x)	K	0.5K	0.3K	0.1K

$\sum p(x) = 1.$

1) The random variable x has the probability distribution shown.

1) The random variable X has the probability distribution shown.

x	1	2	3	4	5
$P(X=x)$	$7c$	$5c$	$4c$	$3c$	c

a) find c , b) $P(X \geq 4)$

$c = 1/20$ $\quad$ $1/5$

Expected value : $E(X) = \sum x \cdot p(x)$

Expectation ← mean (μ).

Ex $\quad$ X - random variable. number sixes obtained when a dice rolled three times. find expected value of X.

Soln. $E(X) = \sum x \cdot p(x)$

$$= 0 \times \frac{125}{216} + 1 \times \frac{25}{72} + 2 \times \frac{5}{72} + 3 \times \frac{1}{216}$$

$$= \frac{25}{72} + \frac{10}{72} + \frac{3}{216} = \frac{36}{72} = \frac{1}{2}$$

$E(X) = \frac{1}{2} = 0.5$

Binomial distribution : (Success , failure).

getting six = $1/6$. $\quad p = 1/6$

Not getting six $= 1 - \frac{1}{6} = \frac{5}{6}$; $\quad q = 5/6$

$$\boxed{p + q = 1}$$

Ex X is a binomially distributed discrete random variable which represents the number of success in six trials. the probability of success in each trial is $1/5$, what is the probability of

(a) exactly four successes (b) at least one success

$n = 6$ $p = 1/5$, $q = 1 - \frac{1}{5} = \frac{4}{5}$

$$P(X=4) = \begin{cases} 1 & \frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{4}{5} \quad - - - - - - \\ 2 & \frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{1}{5} \\ 3 & 4/5 \;\; 4/5 \;\; \frac{1}{5} \;\; 1/5 \;\; 1/5 \;\; 1/5 \;\; 1/6 \\ 4 & \end{cases}$$

$${}^6C_4 \times \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^2$$

${}^6C_r$

$$\boxed{P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}} \longleftarrow \text{Binomial distribution.}$$

$$P(X=4) = \binom{6}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2 =$$

$$\binom{6}{4} = {}^6C_4$$

$$P(X=0) = \left(\frac{4}{5}\right)^6 = \binom{6}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{6-0}$$

$$P(X=6) = \left(\frac{1}{5}\right)^6 = \binom{6}{6}\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^{6-6}$$

$${}^6C_0 = 1.$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}^6C_6 = \frac{6!}{6!\;0!} = 1.$$

b) $P(X \geq 1) = 1 - P(X=0)$

$$= 1 - \binom{6}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$$

$$= 1 - \left(\frac{4}{5}\right)^6 = \underline{\underline{0.738}}$$

Ex On any morning the probability that Sam takes a bus to work is 0.4. Find the probability that, in a working week of five days, Sam takes the bus to work twice.

Soln

$$P(X=2) = \binom{5}{2}(0.4)^2(0.6)^3$$

$$= 0.346$$

Ex A drug is know to have an 80% success rate for people using it, being cured. A medical testing programme administred this drug to two groups of 10 patients. Find the probability that all 10 patients were cured in both groups.

$X \sim B(n,p) \Rightarrow X \sim B(10, 0.8)$

Soln

$n = 10$, $p = 0.8$, $q = 0.2$

X – the number of patients cured in a grp of 10

$$P(X=10) = \binom{10}{10}(0.8)^{10}(0.2)^0 = (0.8)^{10} = 0.1037$$

$$[P(X=10)]^2 = 0.1037 \times 0.1037$$

$$= 0.0115$$

Ex A box contains a large number of carnations $\frac{1}{4}$ of which are red. The rest are white

carnations are picked at random from the box. How many carnations must be picked so that the probability that there is at least one red carnation picked so that the probability that there is at least one red carnation among them is greater than 0.95?

Sol^n X – number of red carnation.

$X \sim B(n, 0.25)$, $q = 0.75$ $P(\text{atleast}) = 1 - P(\text{none})$

$P(X \geq 1) > 0.95$

$1 - P(X=0) > 0.95$

$1 - \binom{n}{0}(0.25)^0(0.75)^n > 0.95$

$\Rightarrow 1 - (0.75)^n > 0.95$

$\Rightarrow 1 - 0.95 > (0.75)^n$

$\Rightarrow 0.05 > (0.75)^n$

$\Rightarrow \log 0.05 > n \log 0.75$

$\Rightarrow n > 10.4$

$\boxed{n = 11}$