Sampling

Thursday, March 4, 2021 5:05 PM

6.5

2. X is a random variable with mean 42.5 and variance 23.1.

a) Find the variance of the mean of a sample of 15 independent observations of X.

b) What is the minimum size of sample needed in order that the variance of the sample mean is less than 1?

$Var(\bar{x}_n) = \frac{23.1}{n} < 1$

$23.1 < n$

$\boxed{n = 24}$

$Var(\bar{x}) = \frac{23.1}{15} = 1.54$

(b) $Var(\bar{x}_n) = \frac{\sigma^2}{n} < 1.$

$\frac{23.1}{n} < 1.$

multiply by n on both side

$n \times \frac{23.1}{n} < 1 \times n$

$\boxed{23.1 < n}$ $n > 23.1$

$n = 24$

1. For the following random variables, work out the mean and variance of the mean of a sample of 10 independent observations of the random variable.

a) $E(X) = 5$; $Var(X) = 4$. mean $= 5$, var $= 0.4$

b) $E(X) = 26.3$; $Var(X) = 7.8$. mean $= 26.3$, var $= 0.78$

c) Z is a random variable with probability distribution given by $P(Z = z) = \frac{5-z}{10}$; $z = 1, 2, 3, 4$.

d) X is a continuous random variable with pdf given by $f(x) = \frac{1}{36}(9 - x^2)$ for $-3 \le x \le 3$.

pdf

z	1	2	3	4
$p(Z=z)$				

$E(Z) = \sum z \cdot p(z) = 2$

$Var(Z) = 1$

$Var(Z) = \sum z^2 \cdot p(z) - \left(\sum z \cdot p(z)\right)^2$

$= 5 - 2^2 = 1$

$E(\bar{Z}_{10}) = 2$

$Var(\bar{Z}_{10}) = 0.1$

(d) $f(x) = \frac{1}{36}(9 - x^2)$

$E(X) = \int_{-3}^{3} x \cdot p(x)\, dx$

$= \int_{-3}^{3} x \cdot \frac{(9-x^2)}{36}\, dx$

$= \frac{1}{36} \int_{-3}^{3} (9x - x^3)\, dx$

$= \frac{1}{36} \left[\frac{9x^2}{2} - \frac{x^4}{4} \right]_{-3}^{3}$

$= \frac{1}{36}(0) = 0$

Σ discrete $\int$ continuous

even function

$f(-x) = f(x)$

$E(X^2) = \int x^2 \cdot p(x)\, dx$

$= 1.8$

$Var(X) = 1.8 - 0 = 1.8$

$E(\bar{X}_{10}) = 0$

$Var(\bar{X}_{10}) = 0.18$

3. A fair spinner is equally likely to land on any of four sections. The sections score 1, 4, 6 and 9 respectively.

a) Find the mean and variance of the score obtained on a single spin.

A random sample of 12 spins is taken and $\bar{X}_{12}$ denotes the mean of the 12 scores obtained.

b) Find the mean and variance of $\bar{X}_{12}$.

Ex 6.6

1. The contents of bottles of water are normally distributed with mean 600 millilitres and standard deviation 7.2 millilitres.

a) Give the distribution of the mean content of a random sample of 6 bottles.

b) Find the probability that the mean content of a random sample of 6 bottles is less than 597 millilitres.

$X \sim N(600, 7.2^2)$

(a) $\bar{X}_6 \sim N\left(600, \frac{7.2^2}{6}\right)$

(b) $P(\bar{X}_6 < 597) = 0.154$

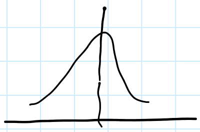
2. Packets of biscuits are labelled as containing 350 grams. The actual contents can be modelled by a normal distribution with mean 352 grams and standard deviation 4.5 grams.

a) Find the probability that the mean contents of a random sample of 10 packets is at least 350 grams.

b) What is the smallest size of random sample for which there is probability of under 1% that the mean contents is under 350 grams?

$\bar{X}_{10} \sim N\left(352, \frac{4.5^2}{10}\right)$

(b) $P(\bar{X}_n < 350) < 0.01$



$\frac{350-352}{\frac{4.5}{\sqrt{n}}} < -2.326$

$\Rightarrow \frac{-2\sqrt{n}}{4.5} < -2.326$

$\Rightarrow \frac{2\sqrt{n}}{4.5} > 2.326$

$\Rightarrow \sqrt{n} > \frac{2.326 \times 4.5}{2}$

$\Rightarrow n > 27.38$

smallest sample size = 28

3. $X \sim N(\mu, \sigma^2)$. $\bar{X}_n$ is the mean of a random sample of n independent observations of X, and $P(|\bar{X}_n - \mu| > 0.25\sigma) < 0.1$.

a) Find the smallest possible value of n.

b) For this value of n find $P(|\bar{X}_n - \mu| < 0.1\sigma)$.

mean $(\bar{X}_n)$ is μ

Y $(\bar{X}_n - \mu)$

mean = 0

var. = $\frac{\sigma^2}{n}$

X is 5

$Y = X - 2$

$E(X) - 2$

$5 - 2 = 3$

$z = ax \pm by$

$var(z) = a^2 var(x) + b^2 var(y)$

$z = ax \pm b$

$var(z) = a^2 var(x)$

$\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\boxed{P(|\bar{X}_n - \mu| > 0.25\sigma) < 0.1}$

$Y = 1\bar{X}_n - \mu$

$E(Y) = E(\bar{X}_n) - \mu$
$= \mu - \mu$
$= 0$

$Y \sim N\left(0, \frac{\sigma^2}{n}\right)$

$var(Y) = 1^2 var(\bar{X}_n)$
$= var(\bar{X}_n)$

$P\left(Z = \left|\frac{(\bar{X}_n - \mu) - 0}{\frac{\sigma}{\sqrt{n}}}\right| > \left(\frac{0.25\sigma - 0}{\frac{\sigma}{\sqrt{n}}}\right)\right) < 0.1$

$P\left(Z = \left|\frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}}\right| > 0.25\sqrt{n}\right) < 0.1$

$|Y| > a$

$Y < -a$

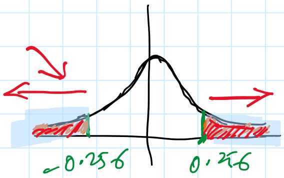
$\frac{\;}{\sqrt{n}}$

$P(|Y| > 0.25\sigma) < 0.1$

$P(Y > 0.25\sigma \;\&\; Y < -0.25\sigma) < 0.1$

$|Y| > a$

$Y > a$ or $Y < -a$



$P(Y < -0.25\sigma) < 0.05$

$$P\left(z\text{-score} = \frac{-0.25\sigma - 0}{\frac{\sigma}{\sqrt{n}}}\right) < 0.05$$

$$\frac{-0.25\cancel{\sigma}\sqrt{n}}{\cancel{\sigma}} < -1.645$$

$$-0.25\sqrt{n} < -1.645$$

multiply by -1

$$0.25\sqrt{n} > 1.645$$

$$\sqrt{n} > \frac{1.645}{0.25}$$

$$n > 43.3$$

$$\boxed{n = 44}$$

(b) $P(|\bar{X}_n - \mu| < 0.1\sigma)$ for $n = 44$

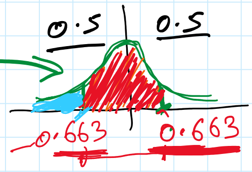
$$(\bar{X}_n - \mu) \sim N\left(0, \frac{\sigma^2}{44}\right)$$

$|Y| < 0.1\sigma$

$$\boxed{-0.1\sigma < Y < 0.1\sigma}$$

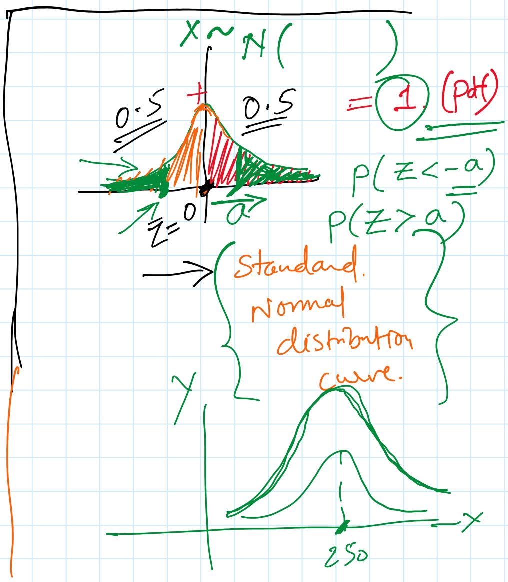
$P(-0.1\sigma < Y < 0.1\sigma) = ?$

z-score standard normal distribution curve



$P(Y < 0.1\sigma) =$

$$P\left(Z_{score} < \frac{0.1\sigma - 0}{\frac{\sigma}{\sqrt{44}}}\right)$$



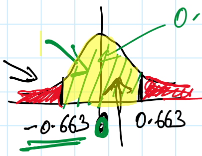
$P(Z_{score} < -0.663)$

$$P\left(Z_{score} < \frac{0.1 - 0}{\frac{6}{\sqrt{44}}}\right)$$

$$= P\left(Z_{score} < 0.1 \times \sqrt{44}\right)$$

$$= P\left(Z_{score} < 0.663\right)$$

$$P(Z_{score} < -0.663)$$



$$1 - 2A = 0.493$$

Given Z = -0.663,

P(x<Z) = 0.25367

P(x>Z) = 0.74633

P(Z<x<0) = 0.24633

P(-Z<x<Z) = 0.49267

P(x<-Z or x>Z) = 0.50733

Z-score, **Z**	-0.663
Probability, **P**(x<Z)	
Probability, **P**(x>Z)	
Probability, **P**(0 to Z or Z to 0)	
Probability, **P**(-Z<x<Z)	
Probability, **P**(x<-Z or x>Z)	

Calculate | Clear

4. The lifetime of Sooperstrong batteries is normally distributed with mean 85 hours and standard deviation 9.2 hours.

a) Find the probability that the mean lifetime of a random sample of 25 Sooperstrong batteries is less than 83 hours.

The lifetime of Powersure batteries is normally distributed with mean 83 hours and standard deviation 2.1 hours.

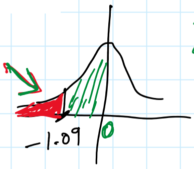
b) Find the probability that the mean lifetime of a random sample of 25 Sooperstrong batteries is shorter than the mean lifetime of a random sample of 5 Powersure batteries.

(a) $S \sim N(85, 9.2^2)$

$\bar{S}_{25} \sim N\left(85, \frac{9.2^2}{25}\right)$

$$P(\bar{S}_{25} < 83) = P\left(Z < \frac{83-85}{\frac{9.2}{\sqrt{25}}} = -1.09\right)$$

$= 0.139$



$\left\{\frac{-1.09}{0}\right\}$

$0.5 - (\ )$

$= 0.138$

(b) $P \sim N(83, 2.1^2)$

$\bar{S}_{25} < \bar{P}_5$

$\bar{S}_{25} - \bar{P}_5 < 0$

$\bar{Y} = \bar{S}_{25} - \bar{P}_5$

$P(\bar{Y} < 0) = ?$

Formula

$Z = X_1 - X_2$

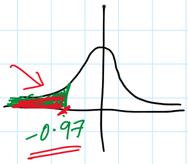
$E(Z) = E(X_1) - E(X_2)$

$Var(Z) = Var(X_1) + Var(X_2)$

$$\bar{Y} \sim N\left(2, \frac{9.2^2}{25} + \frac{2.1^2}{5}\right) = N(2, 4.27)$$

$$P(\bar{Y} < 0) = P\left(Z < \frac{0-2}{\sqrt{4.27}} = -0.97\right)$$

$$P(\bar{Y}<0) = P\left(Z < \frac{0-2}{\sqrt{4.27}} = \underline{-0.97}\right)$$



$$= \underline{0.166}$$

$\underline{6.7 - 5}$

$\boxed{n \geq 30}$

success → $p + q = 1$ ← failure

$X \sim B(10, 0.3)$

n p

mean $= np$, var $= npq$

$= 3$, var $= \underline{2.1}$

(i) $80 \geq 30$) $\bar{X}_{80} \overset{\text{approx}}{\sim} N\left(3, \frac{2.1}{80}\right)$

$\bar{X}_{72} \sim N\left(3, \frac{2.1}{72}\right)$

var

$\sigma = \sqrt{\frac{2.1}{72}}$

(ii) $P(\bar{X}_{72} > 3.25) = P\left(Z > \frac{(3.25-3)\sqrt{72}}{\sqrt{2.1}} = 1.464\right)$

$= \underline{0.0716}$