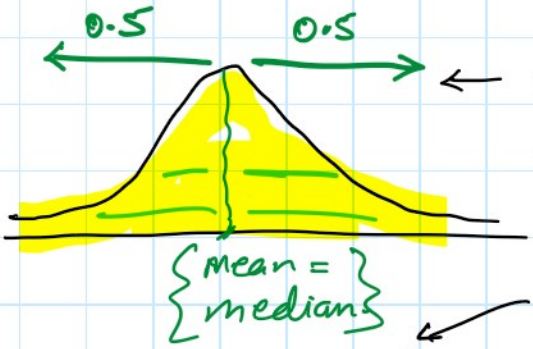
# Normal distribution

Thursday, April 22, 2021 5:50 AM

{ Physical attributes of people, time, exam scores.

Continuous random variable.

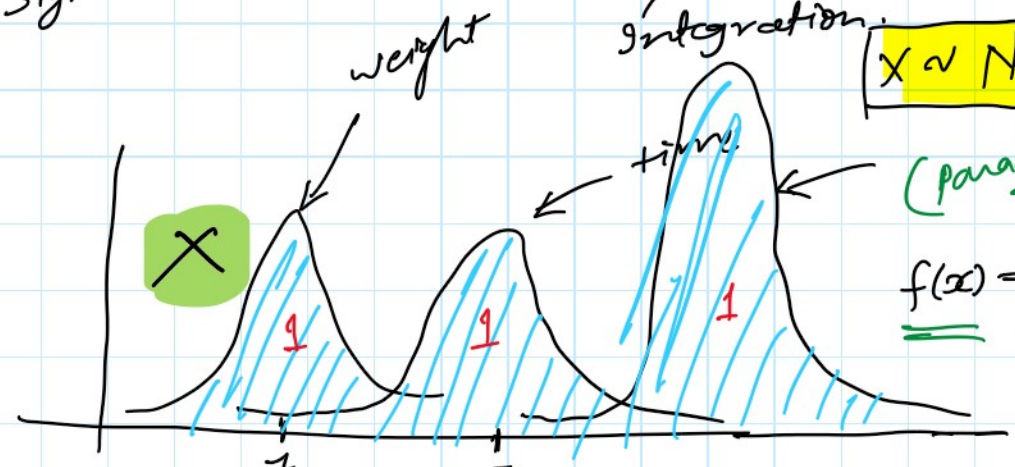



discrete random. → $\sum p = 1$. (Sigma)

continuous. → $\int f(x) = 1$. ← Total probability

integration.

$X \sim N(\mu, \sigma^2)$ (parameter: mean, variance)




$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$

$-\infty < x < \infty$

$Z = \frac{X-\mu}{\sigma}$

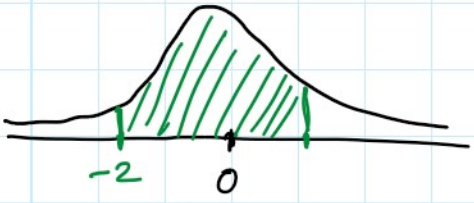
(Z) Standard Normal distribution.

$(\mu = 0, \sigma = 1)$

$Z \sim N(0, 1^2)$

Ex Given that $Z \sim N(0,1)$, sketch the required area under the standard Normal curve and find probability using GDC.

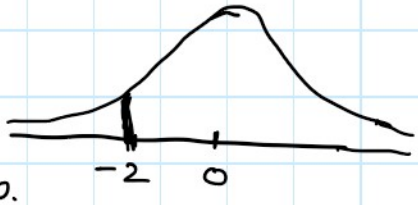
a) $P(-2 < z < 1)$



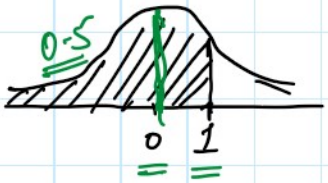

{ probability is same as area under the curve.

$P(-2 < z < 1) = 0.81$

$P(z = -2) = 0$ ← continuous.




b) $P(z < 1) = 0.841$




0.5
0.341
0.841

c) $P(z > -1.5) = 0.933$

d) $P(z < 0) = 0.5$

e) $P(|z| > 0.8) = 0.424$

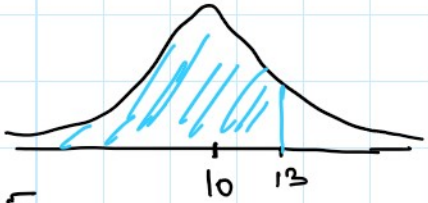
$|z| > 0.8$


$z > 0.8$

$z < -0.8$

Ex Given that $X \sim N(10, 2^2)$. ← variance($\sigma^2$)

Find a) $P(X < 13)$




$X < 13$

$$z = \frac{x - \mu}{\sigma} = \frac{13 - 10}{2} = 1.5$$

$P(X < 13) = P(z < 1.5)$

$= 0.933$

$= 0.933$


LL 0
UL 1.5
$\mu$ 0
$\sigma$ 1

0.433
0.5
0.933

b) $P(X > 9)$ , $X \sim N(10, 2^2)$

$$z = \frac{9-10}{2} = -0.5$$

$$P(X > 9) = P(Z > -0.5)$$

$$= 0.691$$


$P(Z > -0.5)$

$= 1 - P(Z < -0.5)$

$=$

c) $P(9.1 < X < 10.3)$ , $X \sim N(10, 2^2)$

$$z = \frac{9.1-10}{2} \quad , \quad z = \frac{10.3-10}{2}$$

$$P(9.1 < X < 10.3) = P(-0.45 < z < 0.15)$$

$$= 0.233$$

**Ex** Eggs laid by a chicken are known to have masses which are N.d. with mean 55g & S.d 2.5 g

find the probability that a single egg laid by this chicken. has mass $M \sim N(55, 2.5^2)$

a) greater than 59g $P(M > 59) = 0.0548$

b) less than 53g $P(M < 53) = 0.212$

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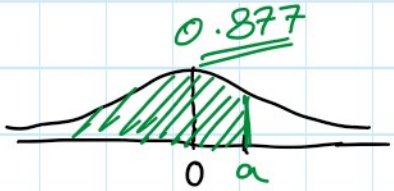
c) between 52g & 54g. $P(52 < M < 54) = 0.230$

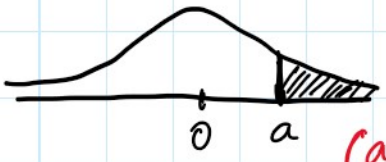
Ex a) $Z \sim N(0,1)$

$P(Z < a) = 0.877$ ← area




$a = 1.16$

b) $P(Z > a) = 0.2$

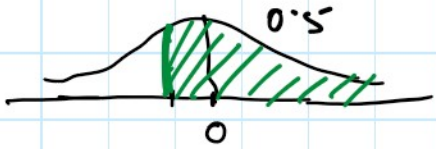



$a = 0.842$

$P(Z < a) = 0.8$ ← percentile method. (area towards left)

c) $P(Z > a) = 0.55$

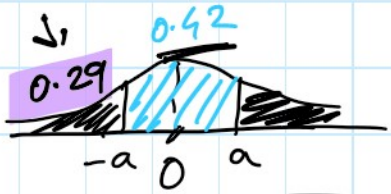
$a = -0.126$




d) $P(-a < Z < a) = 0.42$

$a = 0.553$

$\frac{(1 - 0.42)}{2} = 0.29$




$P(Z < a) = 0.71$

$P(Z < -a) = 0.29$

$P(Z < a) = 0.71$

$\mu = 0, \sigma = 1$

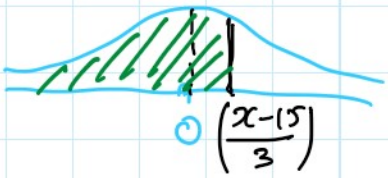
Ex Given that $X \sim N(15, 3^2)$, find the value of $X$ for which

$P(X < x) = 0.75$

$P(\bar{X} < x) = 0.75$


↓ standard Normal distribution.

$$z = \frac{x-\mu}{\sigma} = \left(\frac{x-15}{3}\right)$$




$$P\left(z < \frac{x-15}{3}\right) = (0.75)$$

apply InvNorm find corresponding Z-score.

$$\left(\frac{x-15}{3}\right) = 0.674$$

$$x = 15 + 3 \times 0.674$$

$$x = 17.02$$

Ex Sacks of potatoes are packed by an automatic loader. The mean mass of sack 5 kg

In test it was found that 10% of bags were over 5.2 kg.

Given that masses of sacks are N.d.

find Standard deviation

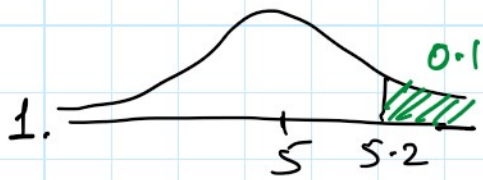
$$M \sim N(5, \sigma^2)$$

$$P(M > 5.2) = 10\% = 0.1$$

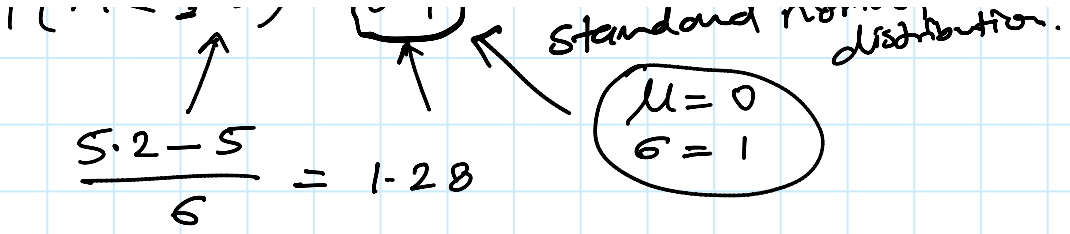
$$P(M > 5.2) = 0.1$$

Left area.

$$P(M < 5.2) = 0.9$$




Standard normal distribution.




$$\Rightarrow \frac{0.2}{\sigma} = 1.28$$

$$\Rightarrow \sigma = \frac{0.2}{1.28} = \underline{0.156} \quad (3 \text{ s.f.}).$$