# Confidence interval

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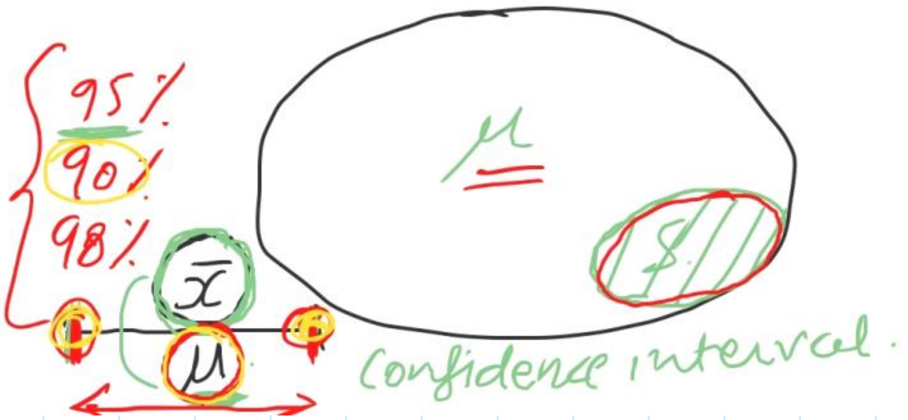
**population:** mean, variance.

mean: ($\mu$) $\sigma^2$

**sample:** mean ($\bar{x}$), variance ($s^2$)

$$\boxed{\bar{X} = \mu}$$

variance: (n data) $s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$



7.1 :- Confidence intervals for the mean of a normal distribution :-

CLT: $n \geq 30$

variance of sample $\bar{x}$ : $\frac{\sigma^2}{n}$

standard deviation $\left(\frac{\sigma}{\sqrt{n}}\right)$

Standard error $\left(\frac{\sigma}{\sqrt{n}}\right)$

standar margin error. $t \cdot \frac{\sigma}{\sqrt{n}}$

$$P\left(\mu - 1.96\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

1) $\left(\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}\right)$ is 90% confidence interv

$\mu$

2) $\bar{x} \pm 2.326 \frac{\sigma}{\sqrt{n}}$ is the 98% confidence interval.

3) $\bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}}$ is the 99% confidence interval.

Q The weights, in grams of bags of cocoa beans are known to have a st devi of 6 grams. A random sample of bags is weighed with the following results.

758, 748, 749, 752, 757, 760, 751, 745, 759, 761

Calculate a 95% conf Int (CI) for the mean weight of a bag of cocoa beans.

$\bar{x} = 754$, $\sigma = 6$ gms.

$\bar{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right) \Rightarrow 754 \pm \left(1.96 \times \frac{6}{\sqrt{10}}\right)$

CI $(750.3, 757.7)$

Confidence interval for the mean of a large sample from any distribution:

large random sample ($n \geq 30$)

95% CI $\left(\bar{x} - 1.96\frac{s}{\sqrt{n}}, \bar{x} + 1.96\frac{s}{\sqrt{n}}\right)$

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

z-score for the required level of confidence.

Q The weight $x$ in gms of a R.S. (random sample) of 50 hamsters were recorded

$\sum x = 10003.3$, $\sum x^2 = 2002665$

calculate 95% CI for the mean weight of hamsters.

Soln

$n \geq 30 \quad (n = 50)$

$\bar{x} = \frac{10003.3}{50} = 200.07$

$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right) = 27.44$

$s = 5.24$

95% CI.

$\left(\bar{x} - 1.96\frac{s}{\sqrt{n}}, \bar{x} + 1.96\frac{s}{\sqrt{n}}\right)$

198.6, 201.5

Q A machine is supposed to produce metal rod which are 5.7 cm long. A random sample of 100 rods produced by machine

| $x$ | $5.60 \le x < 5.65$ | $5.65 \le x < 5.70$ |
|---|---|---|
| $f$ | 15 | 31 |
| $x$ | $5.70 \le x < 5.75$ | $5.75 \le x \le 5.80$ |
| $f$ | 36 | 18 |

Calculate 95% CI for mean length.

$$S^2 = \frac{1}{n-1}\left(\sum m^2 f - \frac{(\sum m f)^2}{\sum f}\right) = 0.00286$$

$\rightarrow \bar{x} = 5.7035$ , $\sum m^2 f = 3253.218$

CI

$\bar{x} \pm 1.96 \times \frac{0.047808}{\sqrt{100}}$

$(5.694, 5.713)$

For a large random sample n, a 95% CI. for the proportion p,

$p = \left(\frac{3}{5}\right)$

$q = 1 - \frac{3}{5}$
$= 2/5$

3

5 games