[Maximum mark: 6]

The following Venn diagram shows the events A and B, where $P(A) = 0.3$. The values shown are probabilities.
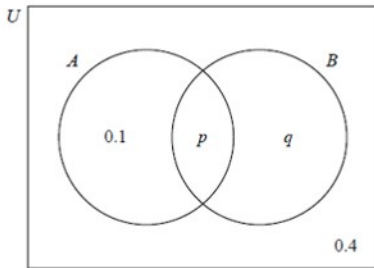
(a) Find the value of p. $0 \cdot 2$ [2]

(b) Find the value of q. $0 \cdot 3$ [2]

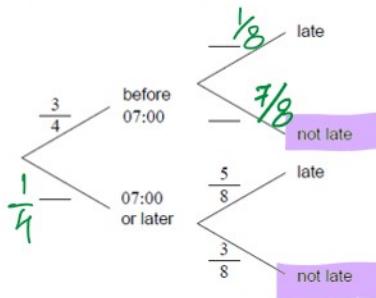(c) Find $P(A' \cup B)$. $0 \cdot 9$ [2]

[Maximum mark: 14]

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$.

If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

(a) **Copy** and complete the following tree diagram.   [3]

$$P(\text{Not late}) = \frac{3}{4} \times \frac{7}{8} + \frac{1}{4} \times \frac{3}{8}$$

$$= \frac{21}{32} + \frac{3}{32} = \frac{24}{32}$$

$$= \frac{3}{4}$$

(b) Find the probability that Pablo leaves home before 07:00 and is late for work.   [2] $\frac{3}{32}$

(c) Find the probability that Pablo is late for work. $\frac{1}{4}$ [3]

(d) Given that Pablo is late for work, find the probability that he left home before 07:00.   [3] $\frac{3}{8}$

$$P(\text{before } 7 \mid \text{late}) = \frac{P(\text{before } 7 \cap \text{late})}{P(\text{late})}$$

(e) Two days next week Pablo will drive to work. Find the probability that he will be late ?   [3]
     at least once.

$$P(\text{at least once} \; \text{late}) = 1 - P(\text{not late twice})$$

$$P(\text{at least one}) = 1 - P(\text{None}).$$

$$= 1 - \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$

$$= 1 - \frac{9}{16} = \frac{7}{16}$$

[Maximum mark: 6]

Two events A and B are such that $P(A) = 0.62$ and $P(A \cap B) = 0.18$.

(a) Find $P(A \cap B')$. $\frac{11}{25} = 0 \cdot 44$ [2]

(b) Given that $P((A \cup B)') = 0.19$, find $P(A \mid B')$.   [4]

$$P(A|\dot B) = \frac{44}{63}$$

[Maximum mark: 15]

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

(a) Find the probability, in terms of n, that the game will end on her

  (i)   first draw; $\frac{2}{n}$

  (ii) second draw. $\frac{n-2}{n} \times \frac{2}{n-1} = \frac{2n-4}{n^2-n}$ [4]

(b) Let $n = 5$. Find the probability that the game will end on her

  (i)   third draw; $\frac{1}{5}$

  (ii) fourth draw. $\frac{1}{10}$ [4]

Hayley plays the game when $n = 5$. She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

$p.$ $\frac{2}{5}$ $\frac{3}{10}$ $\frac{1}{5}$ $\frac{1}{10}$

Number of draws	1	2	3	4
Money earned back (M)	0	20	$8k$	$12k$

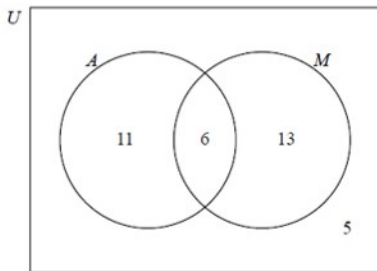(c) Find the value of k so that this is a fair game.   [7]

$$\text{Expectation} = \sum x \cdot p$$

$$\frac{2}{5}(0) + \frac{3}{10} \times 20 + \frac{1}{5} \times 8K + \frac{1}{10} \times 12K = 20.$$

$$\Rightarrow k = 5$$

[Maximum mark: 6]

In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.

(a) Write down the number of students in the group who take art class.   17   [2]

(b) One student from the group is chosen at random. Find the probability that

  (i)   the student does not take art class; $18/35$

  (ii) the student takes either art class or music class, but not both. $24/35$ [4]

[Maximum mark: 6]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.
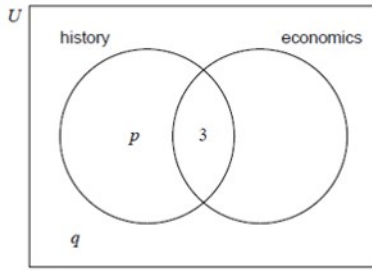
[Maximum mark: 6]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.

U

history   economics

p 3

q
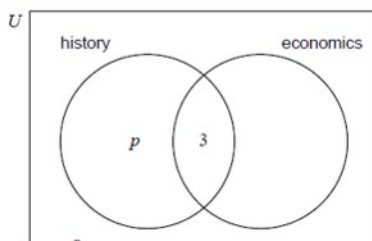
(a)   Find the value of

   (i) p; $= 10$

   (ii) q. $= 2$ [4]

(b)   A girl is selected at random. Find the probability that she takes economics but not history.   [2]
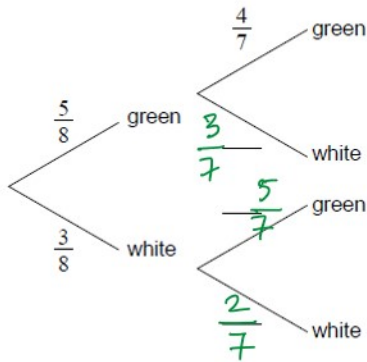
$\frac{1}{4}$

[Maximum mark: 6]

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

(a)   Complete the following tree diagram.   [3]

$\frac{4}{7}$ — green

$\frac{5}{8}$ green $\frac{3}{7}$ — white

$\frac{5}{7}$ — green

$\frac{3}{8}$ white $\frac{2}{7}$ — white

(b)   Find the probability that exactly one of the selected balls is green. $\frac{15}{28}$ [3]

[Maximum mark: 13]

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake. The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

(a)   Find the probability that a customer chosen at random will buy

   (i)   both a sandwich and a cake;

   (ii) only a sandwich. 0.55 [4]

On a typical day 200 customers come to the café.

Binomial

(b)   Find

   (i)   the expected number of cakes sold on a typical day;

   (ii) the probability that more than 100 cakes will be sold on a typical day.   [4]

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

#

(c)   (i)   A customer is selected at random. Find the probability that the customer is male and buys a sandwich. 0.368

   (ii) A female customer is selected at random. Find the probability that she buys a sandwich.   [5]
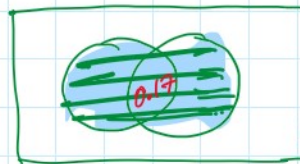
0.72 + 0.45 = 1.17
1.17 − 1 = 0.17

0.17

0.72 − 0.368
= P(S∩F)

(ii) A female customer is selected at random. Find the probability that she buys a sandwich. [5]

$= P(S \cap F)$

$$P(S/F) = \frac{P(S \cap F)}{P(F)} = \frac{0.72 - 0.368}{0.54}$$

$$= 0.652$$

[Maximum mark: 8]

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

(a) Complete the given probability tree diagram for Iqbal's three attempts, labelling each branch with the correct probability. [3]
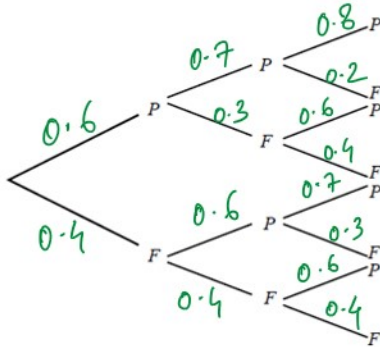
(b) Calculate the probability that Iqbal passes at least two of the papers he attempts. [2]   0.696

(c) Find the probability that Iqbal passes his third paper, given that he passed only one previous paper. [3]

$$P(T/0) = \frac{0.657}{(0.6 \times 0.3 \times 0.6) + (0.4 \times 0.6 \times 0.7)}$$
$$= \frac{(0.6 \times 0.3 \times 0.6) + (0.4 \times 0.6 \times 0.7)}{(0.6 \times 0.3) + (0.4 \times 0.6)}$$

[Maximum mark: 6]

Consider two events, A and B, such that $P(A) = P(A' \cap B) = 0.4$ and $P(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $P(A \cup B)$. [3]

(b) Show that the events A and B are not independent. [3]