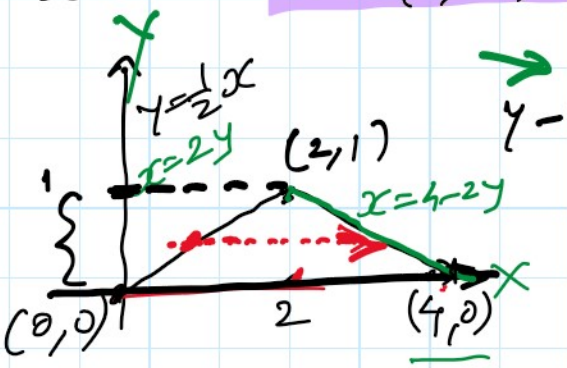
Application of double integral...

Friday, July 24, 2020 11:05 AM

Ex: $\rho(x,y) = x$ $\qquad R = (0,0), (2,1), (4,0)$



$$y - 0 = \frac{-1}{2}(x-4)$$

$$y = -\frac{1}{2}x + 2$$

$$\Rightarrow x = 4 - 2y$$

$$M = \int_{y=0}^{y=1}\int_{x=2y}^{x=4-2y} x \, dx \, dy$$

$dx\,dy$

$$M_x = \iint y\,x \, dx\, dy$$

$$M_y = \iint x \cdot x \, dx \, dy.$$

$$COM = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$\boxed{\begin{array}{c} a \le x \le b \\ c \le y \le d \end{array}}$ — Rectangle

$a \le x \le b$
$0 \le y \le x$ $\Big\}$ $\triangle$

$a \le x \le b$
$0 \le y \le x^2$ — polar.

$0 < y \le x^2$

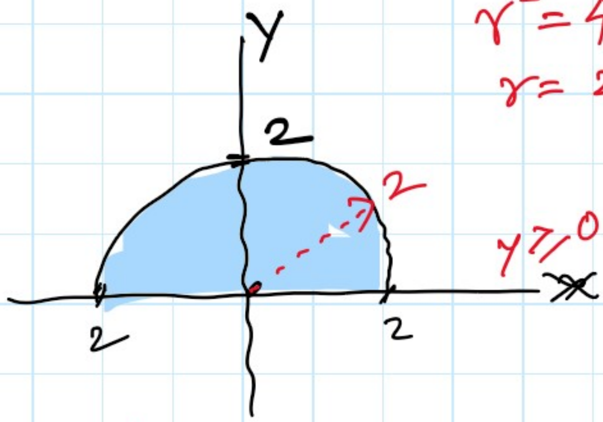
Ex Find total charge of a plate of a charge density function of $\sigma(x,y) = \sqrt{x^2+y^2}$ & charge is spread across $x^2+y^2 \le 4$ & $y \ge 0$.

$$\text{Total charge} = \iint_R \sigma(x,y)\, dA$$

$x^2+y^2 \le 4$

$r^2 = 4$

$r = 2$



$$Q = \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=2} r \cdot \; r\, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^2 d\theta = \int_0^{\pi} \frac{8}{3}\, d\theta = \left[\frac{8\theta}{3}\right]_0^{\pi}$$

$$Q = \frac{8\pi}{3}\ C$$

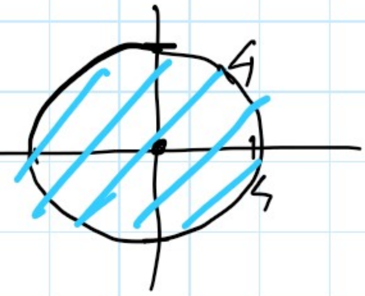
Ex Find Average temperature:-

$T = 400 \cos\left(0.1\sqrt{x^2+y^2}\right)$ ✓ Temperature density f^n

Dimension of the plate is $x^2+y^2 \le 16$

Total temp.

$$\text{Average Temp} = \frac{\text{Total temp.}}{\text{Area}}$$



$$T = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=4} 400\cos(0.1r)\, r\,dr\,d\theta$$

Integration by part

$$T = \underline{19309.066\,°C}$$

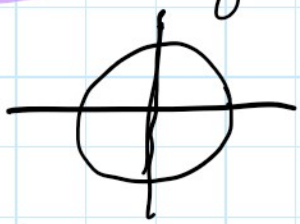
Area $= 16\pi$

$$\text{Avg temp} = \frac{19309.066}{16\pi} = \underline{384.1°C}$$

Ex population density :-

$$\sigma(x,y) = 3000\, e^{-(x^2+y^2)}$$

find out total population of 1 mile of center.



Sol^n

$$\text{population} = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1} 3000\, e^{-r^2}\, r\,dr\,d\theta$$

$$\boxed{\text{Total population} = 3000\pi(1-e^{-1}) \approx \underline{\underline{5958}}}$$

$r=0,\ u=0$ $\Big\}$

$-r^2 = u$

$-2r\,dr = du$

$$\left.\begin{array}{l} r=0, \; u=0 \\ r=1, \; u=-1 \end{array}\right\}$$

$$-2r\,dr = du$$

$$r\,dr = -\frac{1}{2}\,du$$

$$P = \int_{\theta=0}^{\theta=2\pi} \int_{u=-1}^{u=0} \frac{1}{2} \times 3000\, e^{u}\, du\, d\theta$$

$$= \int_{0}^{2\pi} \left[\frac{1}{2} \times 3000 \times e^{u} \right]_{-1}^{0}$$

$$= \int_{0}^{2\pi} \frac{3000}{2} \times \left(1 - e^{-1}\right) d\theta.$$

$$= \frac{3000}{2} \left(1-e^{-1}\right) \left[\theta\right]_{0}^{2\pi}$$

$$= \frac{3000}{\cancel{2}} \left(1-e^{-1}\right) \cdot \cancel{2}\pi$$

$$= 3000\pi\left(1-e^{-1}\right) \approx 5958$$