Probability

1) Experimental Probability
2) Theoretical Probability. (0.5)

$$T \quad T \qquad P(H) = \frac{0}{2} = 0$$

$n=2$

$\rightarrow$ Increase the number of trial $\rightarrow$

$\underline{n=10}$ $\qquad \frac{3}{10} = 0.3$

$n=100$ $\qquad \frac{45}{100} = 0.45$ $\Big\}$ Experimental $\xrightarrow[\text{to}]{\text{tends}}$ theoritical prob

Monty Hall problem! (80°).

Car 1   pick.   2, 3

	1	2	3
	D_1	D_2	D_3

R 1)   3   2 ↑   stick to original — car.

R 2)   3   2   car. (switch)

R 3)   3   2   Loss (don't switch).

$\frac{2}{3}$ $\qquad \frac{1}{3}$ $\qquad \frac{1}{2}$

①

$$P(c) = \frac{1}{3}$$

② [1]   [2]   [▨]   [10]

original pick.

open ⑧ door $\frac{9}{10}$

$\frac{1}{10}$
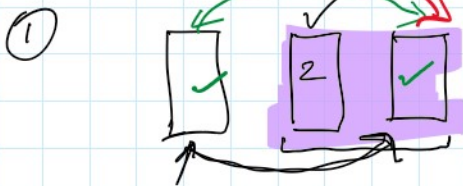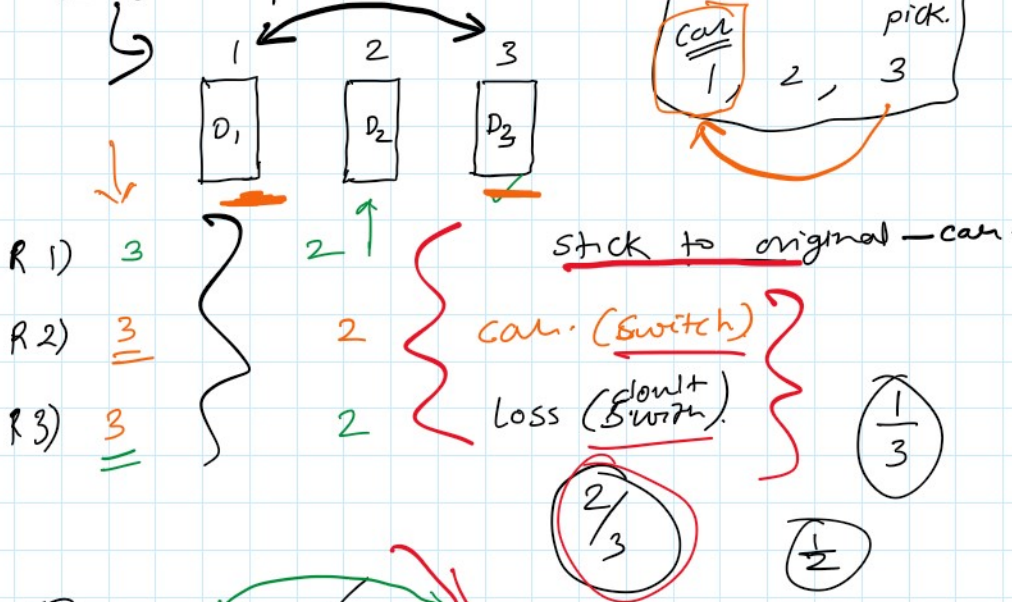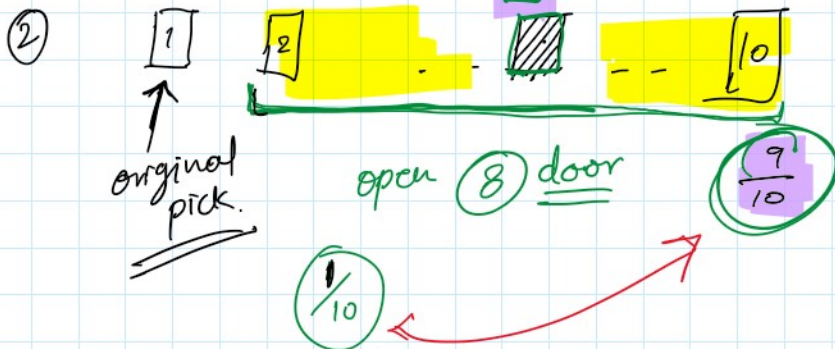
① Two unbiased dice are rolled. Event A is that the numbers on the upper face are equal and event B is that both numbers are odd.
Find these probabilities:
ⓐ P(A)
ⓑ P(B)
ⓒ P(A ∩ B)
ⓓ P(A ∪ B)

P(A and B) ← occurrence of both the event together. ∩

P(A or B) ∪

1 coin → 2 (H,T)
2 coins → 2 × 2
        = 4
[(H,H), (H,T), (T,H), (T,T)]

3 coins ∴ 8

- HHH
- HHT
- HTH
- THH
- HTT
- THT
- THH
- TTT

dice   (1, 2, 3, 4, 5, 6.)

2 dice
= 6 × 6
= 36.

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1)   (2,6)
(3,1)   (3,6)
(4,1)   (4,6)
(5,1)   (5,6)
(6,1)   (6,6)

$$P(A) = \frac{\text{No of favourable event}}{\text{Total No. of events.}} = \frac{6}{36} = \frac{1}{6}$$
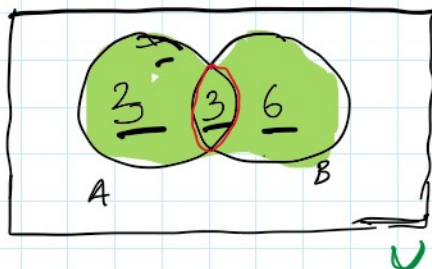
$$P(B) = \frac{9}{36} = \frac{1}{4}$$

$$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

P(A ∪ B) =
|
union.

$\left\{\right. 9$
1. { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) }

{ (1,1), (1,3), (1,5), (3,1), (3,3), (3,5)
  (5,1), (5,3), (5,5), }

A   3 | 3 | 6   B
venn diagram.

$$P(A \cup B) = \frac{12}{36} = \frac{1}{3}$$

either (A) or (B).

(addition rule) **Inclusion - Exculsion formula.**

✓ $\boxed{P(A) + P(B) - \underline{P(A \cap B)} = P(A \cup B)}$

Intersection   Union.

$$P(A \cup B) = \frac{1}{6} + \frac{1}{4} - \frac{1}{12}$$

$$\frac{1}{3} = \frac{2+3-1}{12} = \frac{4}{12} = \frac{1}{3}.$$

A fair coin is tossed three times.
a Find the sample space U.
b Hence write down the probability of obtaining exactly one tail.

Sample Space:

a) $n(S) = 8$

— HHT
— HTH
— THH

b) $P(E) = \frac{3}{8}$ ←

A fair 20-sided dice with faces numbered from 1 to 20 is rolled.

The event M is defined as "the number obtained is a multiple of 3".

a Determine $P(M)$.

The dice is rolled 100 times.

b How many times would you expect to get a multiple of 3?
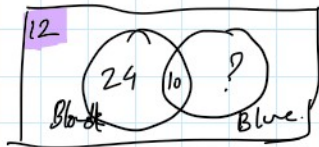
$$P(M) = \frac{6}{20}.$$

$$P(M) = \frac{1}{2}$$

$$\frac{3}{10} \times 100 = \underline{\underline{30}}$$

$$n = \boxed{100}$$

Expectation $E(H) = 100 \times \frac{1}{2} = 50 = \underline{\underline{n \cdot P(H)}}$

9 In a group of 50 students, 10 have blue eyes and blond hair and 12 have neither blue eyes nor blond hair. If the total number of students with blond hair is 34, find the total number of students with blue eyes.

12

24 10 ?
Black   Blue

$$50 - (24 + 12) = 50 - 36$$
$$= 14$$

The colours of cars passing the school gate one morning are given in the table.

Colour	Frequency
Red	26
Black	18
White	20
Green	12
Yellow	3
Blue	16
Other	15
Total	**110**

✓

a Estimate the probability that the next car to pass the school gates will be red.

$26/110$

b Using this information, if 150 cars were to pass the school gate the next day, estimate the number of red cars expected that morning.

$150 \times \dfrac{26}{110}$

The colour of cars is also recorded on the following day. The results are given in the table.

Colour	Frequency
Red	20
Black	21
White	12
Green	9
Yellow	4
Blue	16
Other	10
Total	**92**

c What is an improved estimate of the probability of the next car to pass the school gate being red?

$$\frac{26 + 20}{110 + 92} = \frac{46}{202}$$

In a group of 50 people, 29 like eating fish (F) and 38 like eating chips (C) and 9 people like neither fish nor chips. How many like both fish and chips?

1 Draw a Venn diagram to show this information.

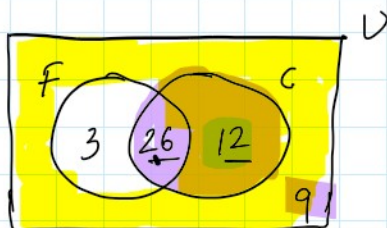**2** Use your diagram to find the probability that a person chosen at random from the group likes:

 a chips
 b both fish and chips
 c fish but not chips. ✓

3 a Describe in words those people in the set $F \cup C$.

 b Find $n(F \cup C)$.

C, C'

$$\boxed{P(C) + P(C') = 1.}$$

$P(C)$

$$\boxed{P(C') = 1 - P(C)}$$

$$P(C) = \frac{38}{50} \quad , \quad P(F \cap C) = \frac{26}{50}$$

$$P(F \cap C') = \frac{3}{50}$$

3 (b) $\quad n(F' \cup C) = 12 + 9 + 26 = 47$

$$n(F' \cup C) = \frac{n(F') + n(C) - n(F' \cap C)}{}$$
$$= (12+9) + (26+12) - 12$$
$$= 12+9+26 = 47$$

At New Blue Bay International School, 15% of the diploma students take Mathematics HL, 20% of these students also take Physics HL and 21% take Physics HL but not Mathematics HL.
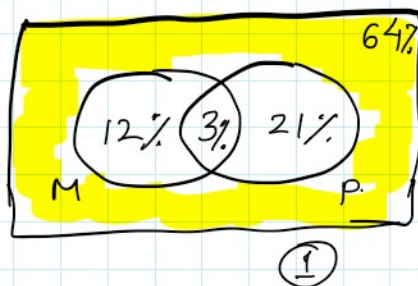If you select a diploma student from this school, what is the probability that the student takes
a   both Mathematics HL and Physics HL
b   neither Mathematics HL nor Physics HL.

(a) $\quad P(M \cap P) = 20\%$ of 15%

$\checkmark$
$$= \frac{20}{100} \times \frac{15}{100}$$
$$= \frac{3}{100} = 3\%$$

$P(M) = 15\% = \frac{15}{100}$

(b) $\quad P(M' \cap P')$
$$= 1 - (0.12 + 0.03 + 0.21)$$
$$= 0.64 = 64\%$$

1. In a group of 38 students, 29 play computer games, 10 play board games and 9 play both.
   a. Draw a Venn diagram to represent this situation.

   A student is selected at random.

   b. Find the probability that the student plays neither computer games nor board games.

2. In a sports club, 40 members play badminton, 37 members play squash, 21 play both and 7 play neither.
   a. Represent this information in a Venn diagram.
   b. Determine how many people are members of the club.
   c. Determine the probability that a member:
      i. plays badminton
      ii. plays both sports
      iii. plays neither sport
      iv. plays at least one sport.

3. A group of 50 people were asked whether they gave their partner a card or a present on their last birthday. The results were: 31 gave a card, 40 gave a present and 25 gave both a card and a present. If one of the people was chosen at random, determine the probability that they gave:
   a. a card or a present
   b. a card but not a present
   c. neither a card nor a present.

4. Set A contains letters needed to spell the word PROBABILITY and set B contains the letters needed to spell the word COMPLEMENTARY.
   a. Draw a Venn diagram for the two sets A and B.
   b. What is in the intersection of A and B?
   c. What is in the union of A and B?

5. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 4, 6, 8, 10} and B = {3, 6, 9}, list the members of the following sets:
   a. $A \cap B$
   b. $A \cup B$
   c. A'
   d. $A' \cap B$
   e. $A \cup B'$
   f. $A' \cup B'$

6. The universal set U is defined as the set of positive integers less than or equal to 15.

   M is the set of integers that are in set U and are multiples of 3.

   F is the set of integers that are in set U and are factors of 30.
   a. List the elements of:
      i. M
      ii. F
   b. Place the elements of M and F in the appropriate regions of a Venn diagram.
   c. A number is chosen at random from U. Find the probability that the number is:
      i. both a multiple of 3 and a factor of 30
      ii. neither a multiple of 3 nor a factor of 30.

7. In a town, 10% of the population watch the news at 1 pm, 30% of people watch the news at 6 pm and 40% of people watch the news at 9 pm.

   It is found that 5% watch at both 6 pm and 9 pm, 4% watch at both 1 pm and 9 pm, 3% watch at 1 pm and 6 pm, and 2% of the people watch all three news shows.
   a. Complete a Venn diagram to show this information. For this Venn diagram, you will need three circles, one for each time the news is on.
   b. Find the probability that a person chosen at random from the town:
      i. watches only the news at 9 pm
      ii. watches only the news at 6 pm
      iii. does not watch the news.

1. A ten-sided dice, numbered 1 to 10, is rolled. Calculate the probability that:
   a. The number rolled is a prime number.
   b. The number rolled is a prime number or a multiple of 3.
   c. The number rolled is a multiple of 3 or a multiple of 4.

2. In a group of 55 tourists, 30 have cameras, 25 are female and 18 are females with cameras. Find the probability that a tourist picked at random from this group is either a camera owner or a female.

3. A letter is chosen at random from the 26-letter English alphabet. Find the probability that the letter is:
   a. in the word MATHEMATICS
   b. in the word TRIGONOMETRY
   c. in the word MATHEMATICS **and** in the word TRIGONOMETRY
   d. in the word MATHEMATICS **or** in the word TRIGONOMETRY.

4. Arnav goes to the library. The probability that he takes out a fiction book is 0.4, a non-fiction book is 0.3 and both a fiction and a non-fiction book is 0.2.
   a. What is the probability that Arnav takes out a fiction book, a non-fiction book or both?
   b. What is the probability that Arnav does not check out a book?

5. If X and Y are two events such that $P(X) = \frac{1}{4}$ and $P(Y) = \frac{1}{8}$ and $P(X \cap Y) = \frac{1}{8}$, find:
   a. $P(X \cup Y)$
   b. $P(X \cup Y)'$

6. If P(A) = 0.2 and P(B) = 0.4 and $P(A \cup B) = 0.5$, find:
   a. $P(A \cap B)$
   b. $P(A' \cup B)$

7. A and B are two events such that $P(A) = \frac{3}{16}$ and $P(B) = \frac{3}{8}$ and it is known that $P(A \cup B) = 3P(A \cap B)$. Find:
   a. $P(A \cup B)$
   b. $P(A \cup B)'$
   c. $P(A \cap B')$