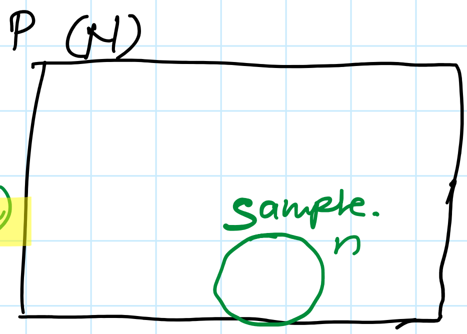
Estimation

Tuesday, February 23, 2021 5:01 PM

1) population $N = 100$
2) sample. $n = 10, 5, 15$
3) Estimation (Unbiased estimation)

mean of population
variance of population.



$n < N$

Eg! The playing time of 9 CDs was recorded. The mean playing time was 58 mins with a variance of 30.4. Estimate the mean and variance of all CDs sold in the country.

Unbiased estimates :-

When using a sample to make estimates about a population, this is said to be an unbiased estimate if the mean of a large number of values taken in the same way is equal to the true value of the parameter.

For a population mean = μ (parameter)

sample mean ($\bar{x}$)

$$\bar{x} = \mu$$

estimator.

Unbiased estimate of population mean:-

$$E(\bar{x}) = \mu = \frac{\Sigma x}{n}$$

Unbiased estimate of population variance:-

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

Σ sigma

$$= \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$s^2 = \frac{n}{n-1} \times \text{sample variance.}$$

Eg1 The playing time of 9 CDs was recorded. The mean playing time was 58 mins with a variance of 30.4. Estimate the mean and variance of all CDs sold in the country.

$$\mu = \bar{x} = 58 \text{ mins}$$

$$s^2 = \frac{n}{n-1} \times \text{sample variance}$$

$$= \frac{9}{8} \times 30.4 \doteq \underline{\underline{34.2}}$$

Eg2 The heights x, in cm, of a random sample of 250 men was measured. $\Sigma x = 43205$ and $\Sigma x^2 = 7{,}469{,}107$. Find unbiased estimates of the population mean and standard deviation.

$$\bar{\mu} = \bar{x} = \frac{\Sigma x}{n} = \frac{43205}{250} = 172.82$$

$$s^2 = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

$$= \frac{1}{249}\left(7469107 - \frac{43205^2}{250}\right) = 9.714$$

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$$s \doteq 3.12$$

For a set of data in grouped intervals you saw in S1 Section 2.3 that the variance was given by $\frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i}$ or $\frac{\sum x_i^2 f_i}{\sum f_i} - \bar{x}^2$. If the data are a sample rather than the full population then you need to adjust as above and use

$$s^2 = \frac{\sum (x_i - \bar{x})^2 f_i}{\left(\sum f_i\right) - 1} = \underline{\left(\frac{n}{n-1}\right)\left\{\frac{\sum x_i^2 f_i}{\sum f_i} - \bar{x}^2\right\}}, \text{ where } n = \sum f_i$$

Eg3

x	4	5	6	7	8
f	3	10	12	15	6

$$\Sigma f = 46$$

$$\mu = \bar{x} = \frac{\Sigma xf}{\Sigma f} = 6.24$$

x_i	f	x_i^2	$x_i^2 f_i$
4	3	16	48
5	10	25	250
6	12	36	432
7	15	49	735
8	6	64	384

$$\Sigma x_i^2 f_i = 1849$$

$$s^2 = \frac{46}{46-1}\left[\frac{1849}{46} - (6.24)^2\right]$$

$$= \frac{46}{45}\left[40.19 - 38.93\right]$$

$$\doteq 1.02(1.2524)$$

$$= \underline{1.277}$$

The weights, x kg, of a random sample of suitcases checked in on a long haul flight are given below. Find estimates of the mean and standard deviation of the weight of suitcases checked in for the flight.

Weight (kg)	$4.5 \le x < 9.5$	$9.5 \le x < 14.5$	$14.5 \le x < 19.5$	$19.5 \le x < 24.5$
Frequency, f	5	9	35	24

x_i	7	12	17	22
f	5	9	35	24

$\Sigma f = 73$

$$\bar{x} = \frac{1266}{73} = 17.34$$

$$s^2 = \frac{1}{n-1}\left(\Sigma x^2 f - \frac{(\Sigma x_i f)^2}{n-1}\right)$$ ✱

$$= \frac{1}{72}\left(23272 - \frac{1266^2}{72}\right)$$

$$= \frac{1}{72}\left(23\cancel{2}272 - 22260.5\right) = \frac{1011.5}{72} = 14.048$$

$$S = \sqrt{14.048} = 3.75$$