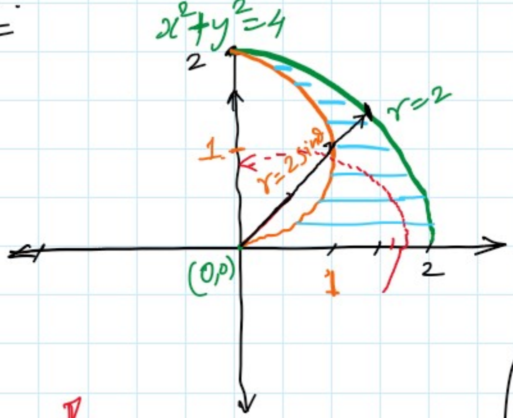
Volume between two surface

Wednesday, July 15, 2020 10:57 AM

Ex: Region $x^2+y^2=4$ & $x^2+y^2=2y$ in Qud 1.

Soln:



$x^2+y^2=4$

$r^2=4$

$r=2$

$x^2+y^2=2y$

$\Rightarrow x^2+y^2-2y=0$

$\Rightarrow x^2+y^2-2y+1=1$

$\Rightarrow x^2+(y-1)^2=1$

$(0,1)$, $r=1$

$r^2-2r\sin\theta=0$

$\Rightarrow r^2=2r\sin\theta$

$\Rightarrow r=2\sin\theta.$

$$V=\int_{\theta=0}^{\theta=\pi/2}\int_{r=2\sin\theta}^{r=2} f(r,\theta)\, r\, dr\, d\theta$$

Ex: volume below $3x+4y+z=12$, bound by region between $x^2+y^2=2x$ & above xy.plane

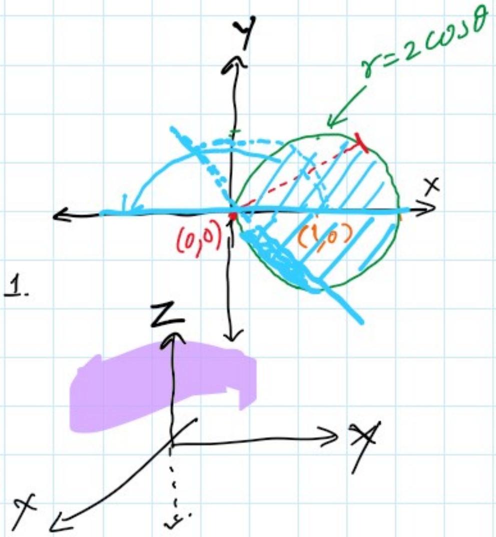
Soln:

$x^2+y^2=2x.$

$\Rightarrow x^2-2x+1+y^2=1$

$\Rightarrow (x-1)^2+y^2=1$

$C(1,0)$, $r^2=1 \Rightarrow r=1.$



$x^2+y^2=2x$

$\Rightarrow r^2=2r\cos\theta$

$\Rightarrow r=2\cos\theta$

$0\le\theta\le\pi/2$

$$2\int_0^{\pi/2} \Rightarrow \int_0^{\pi}$$

$z=f(x,y)$

$z=12-3x-4y$ ← (1,0)

$=12-3(1)-4(0)=9$ (+ve)

$$V=\int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=2\cos\theta}(12-3r\cos\theta-4r\sin\theta)\,r\,dr\,d\theta$$

$z=12-3x-4y$

$=12-3r\cos\theta-4r\sin\theta$

$$=\int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=2\cos\theta} 12r - r^2(3\cos\theta+4\sin\theta)\,dr\,d\theta.$$

$$\int_{\theta=0}^{\theta=\pi}\left[6r^2-\frac{1}{3}r^3(3\cos\theta+4\sin\theta)\right]^{2\cos\theta} d\theta$$

$\theta=0 \quad r=\ldots$

$$= \int_{\theta=0}^{\theta=\pi} \left[6r^2 - \frac{1}{3}r^3(3\cos\theta + 4\sin\theta) \right]_0^{2\cos\theta} d\theta$$

$$= \int_{\theta=0}^{\theta=\pi} 24\cos^2\theta - \frac{8}{3}\cos^3\theta\,(3\cos\theta + 4\sin\theta)\, d\theta$$

$$= \int_{\theta=0}^{\theta=\pi} \underbrace{24\cos^2\theta}_{(A)} - \underbrace{8\cos^4\theta}_{(B)} - \underbrace{\frac{32}{3}\cos^3\theta\sin\theta\, d\theta}_{(C)}$$

u-sub

GCD

4.25c

(A) $12(1+\cos 2\theta)$

(B) $\Rightarrow \int 8\cos^4\theta\, d\theta = \int 8\left[\frac{1+\cos 2\theta}{2}\right]^2 d\theta$

$$= \int 2\,(1 + 2\cos 2\theta + \cos^2 2\theta)$$

$$= \int 2\left(1 + 2\cos 2\theta + \frac{1}{2} + \frac{\cos 4\theta}{2}\right) d\theta$$

$$= \left[2\theta + 2\sin 2\theta + \theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi}$$

$$= 2\pi + \pi = 3\pi \quad \leftarrow B.$$

(A) $\left[12\theta + 6\sin 2\theta\right]_0^{\pi} = 12\pi$

(C) $\int_0^{\pi} \frac{32}{3}\cos^3\theta\sin\theta\, d\theta = \int_{-1}^{1} \frac{32}{3}u^3\, du$

$\cos\theta = u$

$-\sin\theta\, d\theta = du$

$\theta=0,\ u=1$

$\theta=\pi,\ u=-1$

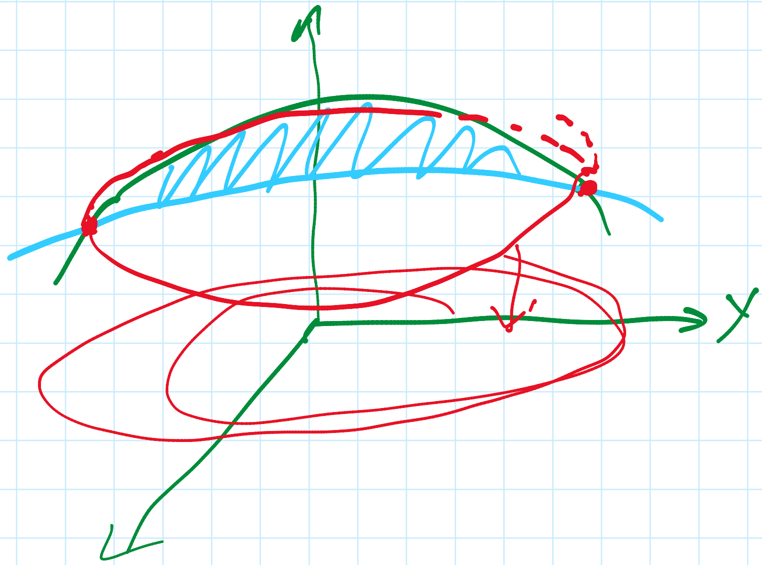
$$= \left[\frac{32}{3} \times \frac{u^4}{4}\right]_{-1}^{1}$$

$$= \frac{8}{3} - \left(\frac{8}{3}\right)$$

$$= 0$$

$$V = 12\pi - 3\pi = \underline{\underline{9\pi}}$$

Ex! Volume between $x^2+y^2+z^2=2$ & $z=\sqrt{x^2+y^2}$


y