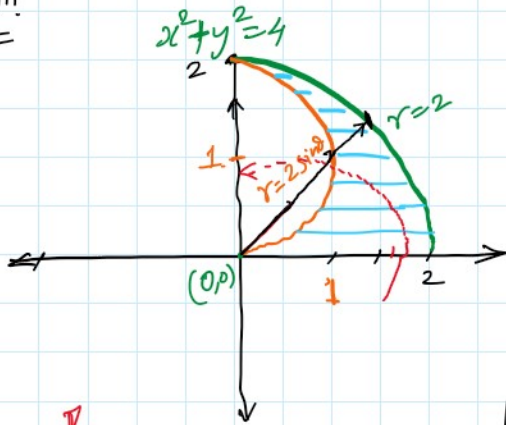


Volume between two surface

Wednesday, July 15, 2020 10:57 AM

Ex: Region $x^2+y^2=4$ & $x^2+y^2=2y$ in Qud.1.

Soln:



$$\begin{aligned} x^2+y^2 &= 4 \\ r^2 &= 4 \\ r &= 2 \end{aligned}$$

$$x^2+y^2=2y$$

$$\begin{aligned} \Rightarrow x^2+y^2-2y &= 0 \\ \Rightarrow x^2+y^2-2y+1 &= 1 \\ \Rightarrow \underline{x^2+(y-1)^2} &= 1 \\ (0,1), r &= 1 \end{aligned}$$

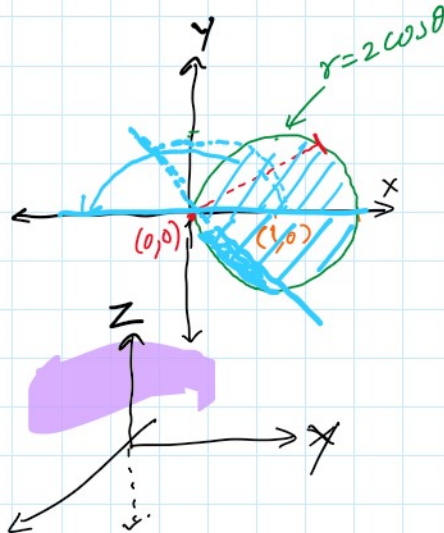
$$\begin{aligned} r^2 - 2r\sin\theta &= 0 \\ \Rightarrow r^2 &= 2r\sin\theta \\ \Rightarrow r &= 2\sin\theta \end{aligned}$$

$$\begin{aligned} \theta = \frac{\pi}{2} \quad r = 2 \\ V = \int \int f(r,\theta) r \, dr \, d\theta \\ \theta = 0 \quad r = 2\sin\theta \end{aligned}$$

Ex: Volume below $3x+4y+z=12$, bound by region between $x^2+y^2=2x$ & above xy-plane

Soln:

$$\begin{aligned} \boxed{x^2+y^2=2x} \\ \Rightarrow x^2-2x+1+y^2 &= 1 \\ \Rightarrow (x-1)^2+y^2 &= 1 \\ C(1,0), r^2=1 &\Rightarrow r=1 \end{aligned}$$



$$\begin{aligned} x^2+y^2 &= 2x \\ \Rightarrow r^2 &= 2r\cos\theta \\ \Rightarrow \underline{r} &= 2\cos\theta \end{aligned}$$

$$\begin{aligned} 0 \leq \theta \leq \frac{\pi}{2} \\ \int_0^{\pi/2} \int_0^{2\cos\theta} \Rightarrow \int_0^{\pi} \end{aligned}$$

$$\begin{aligned} z &= f(x,y) \\ z &= 12-3x-4y \quad \swarrow (1,0) \\ &= 12-3(1)-4(0) = 9 \quad (\neq 0) \end{aligned}$$

$$\begin{aligned} V &= \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=2\cos\theta} (12-3r\cos\theta-4r\sin\theta) r \, dr \, d\theta \\ &= \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=2\cos\theta} 12r - r^2(3\cos\theta+4\sin\theta) \, dr \, d\theta \end{aligned}$$

$$\begin{aligned} z &= 12-3x-4y \\ &= \underline{12-3r\cos\theta-4r\sin\theta} \end{aligned}$$

$$\begin{aligned}
 & \theta=0 \quad r=2 \\
 & = \int_{\theta=0}^{\theta=\pi} \left[6r^2 - \frac{1}{3} r^3 (3\cos\theta + 4\sin\theta) \right]_0^{2\cos\theta} d\theta \\
 & = \int_{\theta=0}^{\theta=\pi} 24\cos^2\theta - \frac{8}{3} \cos^3\theta (3\cos\theta + 4\sin\theta) d\theta \\
 & = \int_{\theta=0}^{\theta=\pi} \underbrace{24\cos^2\theta}_{(A)} - \underbrace{8\cos^4\theta}_{(B)} - \underbrace{\frac{32}{3}\cos^3\theta\sin\theta}_{(C)} d\theta \\
 & \text{① } 12(1+\cos 2\theta) \quad \text{② } \text{GCD } u\text{-sub } 4 \cdot 25^c
 \end{aligned}$$

$$\begin{aligned}
 \text{② } \Rightarrow \int 8\cos^4\theta d\theta &= \int 8 \left[\frac{1+\cos 2\theta}{2} \right]^2 d\theta \\
 &= \int 2(1+2\cos 2\theta + \cos^2 2\theta) \\
 &= \int 2 \left(1 + 2\cos 2\theta + \frac{1}{2} + \frac{\cos 4\theta}{2} \right) d\theta \\
 &= \left[2\theta + 2\sin 2\theta + \theta + \frac{1}{4}\sin 4\theta \right]_0^\pi \\
 &= 2\pi + \pi = 3\pi \quad \leftarrow B.
 \end{aligned}$$

$$\text{③ } \left[12\theta + 6\sin 2\theta \right]_0^\pi = 12\pi$$

$$\begin{aligned}
 \text{④ } \int_0^\pi \frac{32}{3} \cos^3\theta \sin\theta d\theta &= \int_{-1}^1 \frac{32}{3} u^3 du \\
 \cos\theta = u \quad \left(\begin{array}{l} \theta=0, u=1 \\ \theta=\pi, u=-1 \end{array} \right) & \rightarrow \left[\frac{32}{3} \times \frac{u^4}{4} \right]_{-1}^1 \\
 -\sin\theta d\theta = du & = \frac{8}{3} - \left(\frac{8}{3} \right) \\
 & = 0
 \end{aligned}$$

$$V = 12\pi - 3\pi = \underline{\underline{9\pi}}$$

Ex: Volume between $x^2+y^2+z^2=2$ & $z=\sqrt{x^2+y^2}$

