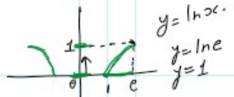
# Solid of revolution

Thursday, September 2, 2021 6:57 AM

**Ex** The graph of $y=\ln x$, $1 \le x \le e$ is revolved through $2\pi$ about $y$-axis. Find the volume of revolution. (Exact ans).

**Soln**




$$V_y = \int_0^1 \pi x^2 \, dy$$

$y = \ln x.$
$x = e^y$
$x^2 = e^{2y}$

$$= \int_0^1 \pi e^{2y} \, dy$$

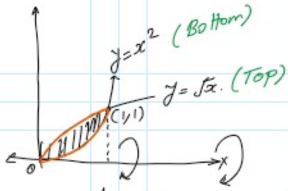
$$= \left[\pi \frac{e^{2y}}{2}\right]_0^1$$

$$= \pi \frac{e^{2(1)}}{2} - \pi \frac{e^{2(0)}}{2}$$

$$= \pi \frac{e^2}{2} - \frac{\pi}{2} = \frac{\pi}{2}(e^2-1) \quad \leftarrow \text{Exact}$$

**Ex** Find the volume of rev. generated by revolving the region between $y=x^2$ & $y=\sqrt{x}$ about the $x$-axis.

**Soln**



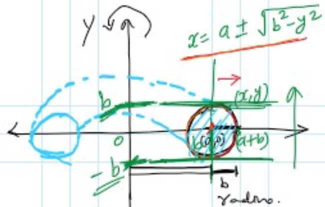

$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx.$$

$$= \pi \int_0^1 x - x^4 \, dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{5} - 0\right]$$

$$= \pi \left[\frac{3}{10}\right] = \frac{3\pi}{10}$$




Top ; Bottom
Right ; left

center $(a,0)$
radius $b$


(locus)
set of points

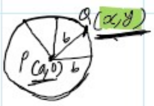
$$V = \pi \int_{-b}^{b} \left(a+\sqrt{b^2-y^2}\right)^2 - \left(a-\sqrt{b^2-y^2}\right)^2 dy$$

$$= \pi \int_{-b}^{b} \cancel{a^2} + \cancel{(b^2-y^2)} + 2a\sqrt{b^2-y^2} - \left(\cancel{a^2} + \cancel{(b^2-y^2)} - 2a\sqrt{b^2-y^2}\right) dy$$

{ Circle is set of ∞ points which are at equidistance from a fixed point (center).

$$= \pi \int_{-b}^{b} 4a\sqrt{b^2-y^2} \, dy$$

$$= 4a\pi \left\{\int_{-b}^{b} \sqrt{b^2-y^2} \, dy\right\}$$





$(x-2)^2+(y-3)^2=25$

distance formula

$PQ = b$

$$\sqrt{(x-a)^2+(y-0)^2} = b$$

$$\Rightarrow (x-a)^2 + y^2 = b^2 \leftarrow \text{eqn of circle.}$$

$x = f^{-1}(y)$

$(x-a)^2 = b^2 - y^2$

$x-a = \pm\sqrt{b^2-y^2}$

$\boxed{x = a \pm \sqrt{b^2-y^2}}$

$y^2 = b^2 - (x-a)^2$

$\boxed{y = \pm\sqrt{b^2-(x-a)^2}}$

$y = f(x)$

Concept of integration or technique of integ.

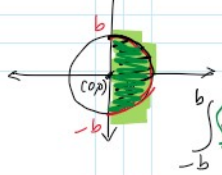
$x = \sqrt{b^2-y^2} \Rightarrow x^2 = b^2-y^2$

$x^2+y^2 = b^2$

$(0,0), b \Longleftrightarrow$ circle

$x = \pm\sqrt{b^2-y^2}$

$x = +\sqrt{b^2-y^2}$

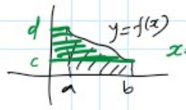



$$\int_{-b}^{b} \sqrt{b^2-y^2} \, dy$$

$$= 4a\pi \times \frac{1}{2}\pi b^2$$

$$V = 2\pi^2 a b^2$$

$a, b$




$x = f^{-1}(y)$

$$\int_c^d f^{-1}(y) \, dy$$

$$= \int_{-b}^{b} \sqrt{b^2-y^2} \, dy$$

$x = +\sqrt{b^2-y^2}$

$x^2 = b^2 - y^2$

c

$= \int_{-b}^{b} \sqrt{b^2 - y^2}\, dy$

$x = +\sqrt{b^2 - y^2}$

$x^2 = b^2 - y^2$

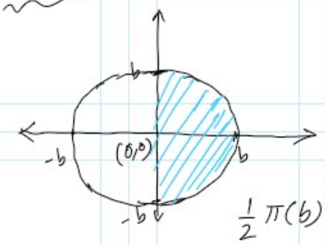
$x^2 + y^2 = b^2$

centre $(0,0)$

radius $b$.

$(x-h)^2 + (y-k)^2 = r^2$

$(h,k)$, $r$




$\frac{1}{2}\pi(b)^2$

$= \frac{1}{2}\pi b^2$