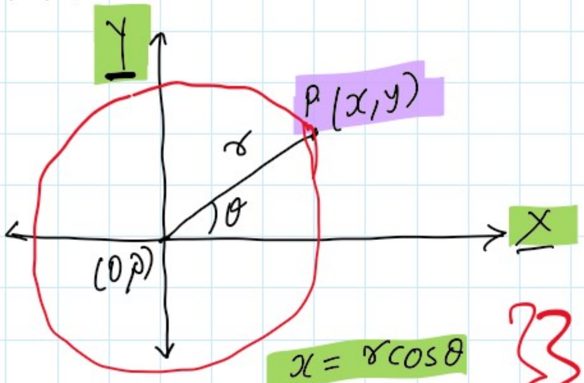
Double integral in polar coordinates

Monday, July 13, 2020 10:54 AM

polar co-ordinates system

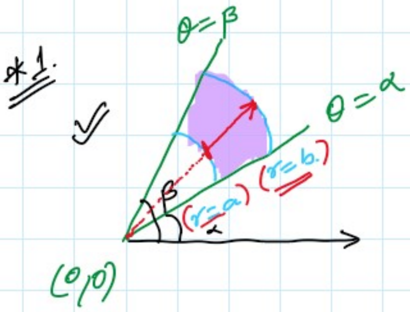


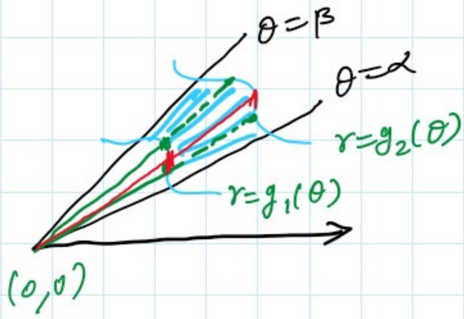
$x = r\cos\theta$

$y = r\sin\theta.$

$x^2+y^2=r^2 \leftarrow$ circle.

*1.





$$\iint_R f(x,y)\, dA.$$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

$\tan\theta = \frac{y}{x}.$

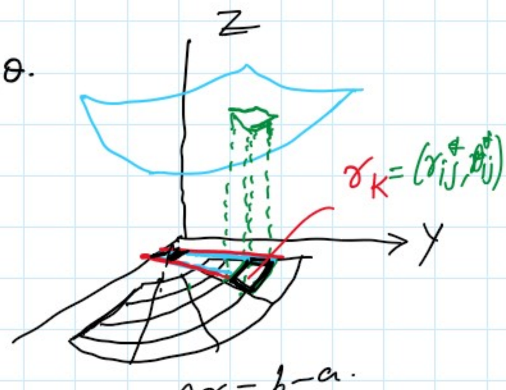
*1)
$$\int_{\theta=\alpha}^{\theta=\beta}\int_{r=a}^{r=b} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

*2.
$$\int_{\theta=\alpha}^{\theta=\beta}\int_{r=g_1(\theta)}^{r=g_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

$$A = \frac{1}{2}b^2\Delta\theta - \frac{1}{2}a^2\Delta\theta.$$

$$= \frac{1}{2}(b^2-a^2)\,\Delta\theta$$

$$= \frac{1}{2}(b+a)(b-a)\,\Delta\theta.$$



$\Delta r = b-a.$

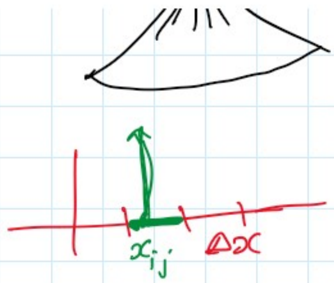
$\Delta\theta.$

Let $A = r_k\, \Delta r\, \Delta\theta$

$(\Delta\theta \to 0)$

$dA = r\, dr\, d\theta.$

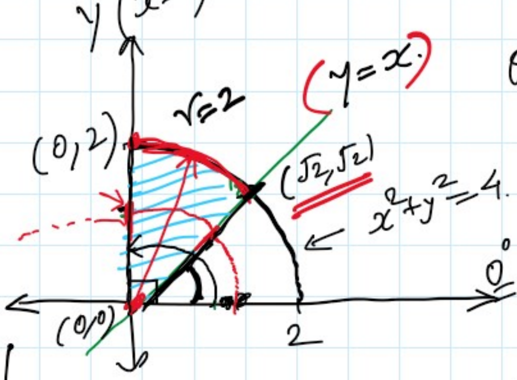
$\boxed{\Delta\theta \to 0}$ $dA = r \, \boxed{dr \, d\theta}$



Ex $\displaystyle\iint_R x y \, dA$

R, the region in (Q1) bound by $x^2+y^2=4$, $x=y$ & $x=0$.

$z = f(x,y) = xy$



$\theta = \tan^{-1}\left(\frac{y}{x}\right)$
$= \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$
$= \tan^{-1}(1) = \frac{\pi}{4}$

$y = x.$

$x^2 + x^2 = 4$
$2x^2 = 4$
$x^2 = 2$
$x = \sqrt{2}$, $y = \sqrt{2}$

$$V = \int_{\theta=\frac{\pi}{4}}^{\theta=\pi/2} \int_{r=0}^{r=2} r\cos\theta \, r\sin\theta \, r \, dr \, d\theta$$

$x^2+y^2=4$
$r^2 = 4 \Rightarrow r = 2$

$\tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2}$

$$= \int_{\theta=\frac{\pi}{4}}^{\theta=\pi/2} \int_{r=0}^{r=2} \left(r^3 \sin\theta\cos\theta\right) dr\, \theta$$

$$= \int_{\theta=\pi/4}^{\theta=\pi/2} \left[\sin\theta\cos\theta \, \frac{r^4}{4}\right]_{r=0}^{r=2} d\theta$$

$$= \int_{\theta=\pi/4}^{\theta=\pi/2} 4\sin\theta\cos\theta \, d\theta.$$

u-sub.

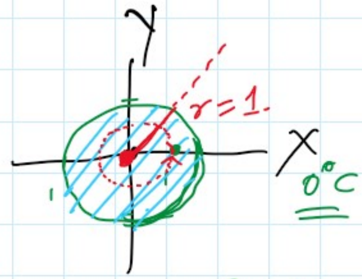
$= 1.$

Ex Find volume between $z = 9 - x^2 - y^2$ &

the xy plane as bound by the cylinder

$x^2+y^2=1.$

$$V=\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1}(9-r^2)\,r\,dr\,d\theta$$



$$z=9-x^2-y^2$$
$$=9-(x^2+y^2)$$
$$=9-r^2$$

$x^2+y^2=1$

$r^2=1$

$r=1$

$$=\int_{\theta=0}^{\theta=2\pi}\left[\frac{9r^2}{2}-\frac{1}{4}r^4\right]_{r=0}^{r=1}d\theta$$

$$=\int_{\theta=0}^{\theta=2\pi}\frac{9}{2}-\frac{1}{4}\,d\theta$$

$$=\int_{\theta=0}^{\theta=2\pi}\frac{17}{4}\,d\theta=\frac{17}{4}[\theta]_{\theta=0}^{\theta=2\pi}=\frac{17(2\pi)}{4}$$

$$=\frac{17\pi}{2}.$$

Ex volume below $z=\frac{y^2}{x^2+y^2}$, above xy-plane & between cylinders $x^2+y^2=1$ & $x^2+y^2=2$.

Solⁿ

$$V=\int_{\theta=0}^{\theta=2\pi}\int_{r=1}^{r=\sqrt{2}}\sin^2\theta\;r\,dr\,d\theta.$$

ANS: $=\frac{\pi}{2}$