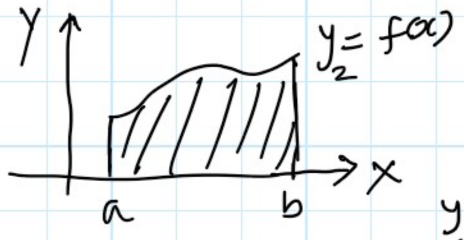
Area under the curve

Thursday, August 26, 2021 6:42 AM

1) $A'(x) = f(x)$

$\Rightarrow A(x) = \int_a^b f(x)\, dx$

2)



$y = 0$

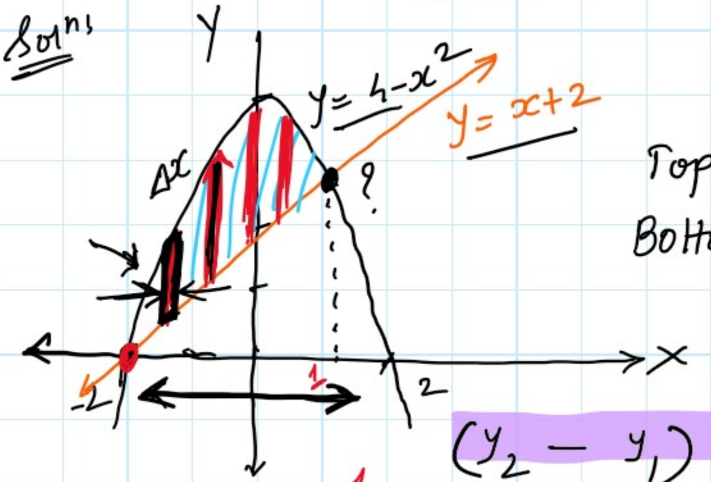
(*) $A = \int_a^b [f(x) - 0]\, dx$

$= \int_a^b f(x)\, dx.$

Area between two curves.

Ex find the area of region bounded by the curves $y = 4 - x^2$ & $y = x + 2$

Sol^n:



Top f^n $y = 4 - x^2$

Bottom f^n $y = x + 2$

$(y_2 - y_1)$ $\Delta x \to 0$

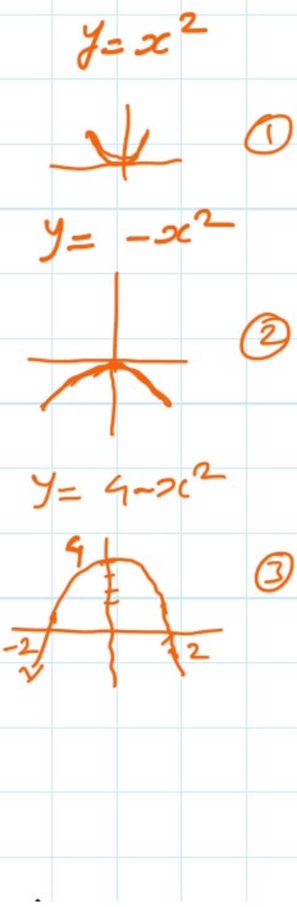
(*) $A = \int_{-2}^{1} [(4 - x^2) - (x + 2)]\, dx.$

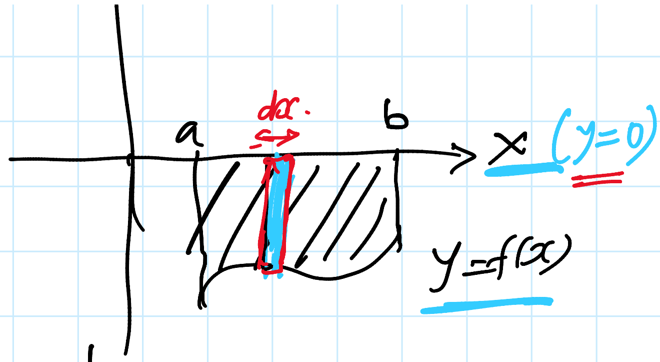
$4 - x^2 = x + 2$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$= 9/2$



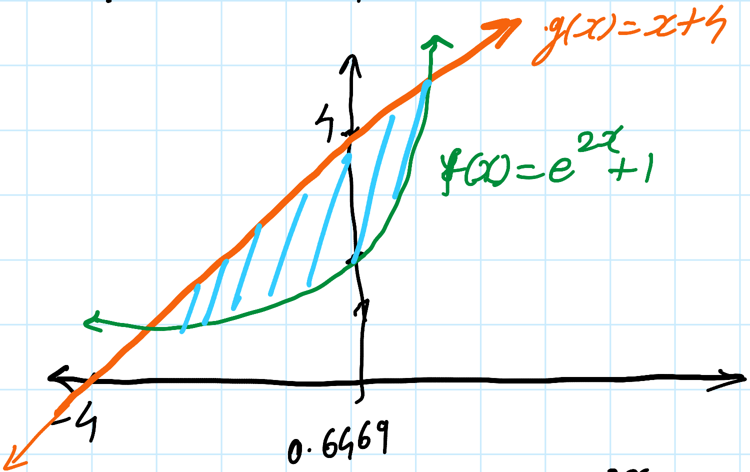


$$A = \int_a^b [0 - f(x)]\, dx$$

$$= \int_a^b -f(x)\ dx.$$

Ex $f(x) = e^{2x} + 1$ & $g(x) = x + 4$ [gDC]

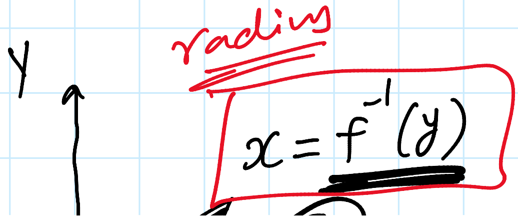
$x = -2.99$, $x = 0.6469$



$$A = \int_{-2.99}^{0.6469} x+4 - e^{2x} - 1\ dx.$$

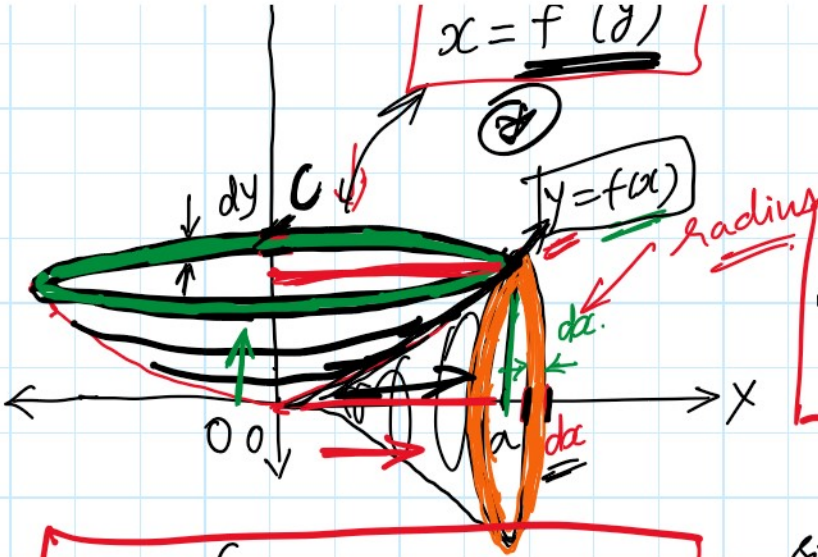
$$= \left[\frac{x^2}{2} + 4x - \frac{e^{2x}}{2} - x \right]_{-2.99}^{0.6469}$$

$$= \underline{\underline{4.83}}$$



$$V = \int_a^b \pi\, [f(x)]^2\, dx$$

$\int_a$



solid of rev. about x axis.

$$V_x = \int_0^a \pi [f(x)]^2 \, dx.$$

$$V_x = \int_0^a \pi y^2 \, dx.$$

solid of revolution about Y axis.

$$V_y = \int_0^C \pi [f^{-1}(y)]^2 \, dy$$

$$V_y = \int_0^C \pi x^2 \, dy$$