


Technique of Integration

Thursday, September 9, 2021 7:14 AM

Indefinite Integration.

① $\int x^n dx = \frac{x^{n+1}}{n+1} + c$



② $\int (2x+5)^3 dx = \frac{(2x+5)^{3+1}}{(3+1) \cdot 2} + c$ X

$\int x^4 dx = \frac{x^5}{5} + c$

$\frac{d}{dx} \left[\frac{x^5}{5} + c \right] = x^4 + 0$

$\frac{d}{dx} \frac{(2x+5)^4}{4 \times 2} = \frac{2(2x+5)^3}{2} = (2x+5)^3$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$

$ax+b$
Linear

$\frac{d}{dx} (\text{RHS}) = \frac{1}{(n+1)a} \cdot (n+1) (ax+b)^{n+1-1} \cdot a$
 $= (ax+b)^n$

Ex $\int (3x+5)^8 dx = \frac{(3x+5)^9}{9 \times 3} + c$

Ex $\int (2x^2+1)^2 dx = \frac{(2x^2+1)^3}{3 \cdot 4} + c$ X

$\frac{d}{dx} (\text{RHS}) = \frac{1}{12} \times 3 (2x^2+1)^2 \cdot 4x$
 $= x(2x^2+1)^2$

$\int (2x^2+1)^2 dx = \int (4x^4 + 1 + 4x^2) dx$
 $= \frac{4x^5}{5} + x + \frac{4x^3}{3} + c$
 $= \frac{4x^5}{5} + x + \frac{4x^3}{3} + c$

$$= \frac{4x^5}{5} + x + \frac{4x^3}{3} + c.$$

$$\int (2x^2+1)^8 dx = ? \quad \text{Trig. Substitution}$$

$$\underline{\underline{\text{Ex}}} \quad \int \frac{1}{x} dx = \int x^{-1} dx. \quad \rightarrow \text{undefined } \times$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

$$\boxed{\int \frac{1}{x} dx = \ln x + c} \quad \oplus$$

$$\underline{\underline{\text{Ex}}} \quad \int \frac{1}{4x+3} dx = \frac{\ln|4x+3|}{4} + c$$

$$\frac{d}{dx} \ln|4x+3| = \frac{1}{4x+3} \times 4 \quad \leftarrow$$

$$\underline{\underline{\text{Ex}}} \quad \int \frac{1}{-x+2} dx = \frac{\ln|-x+2|}{-1} + c$$

$$= -\ln|-x+2| + c$$

$$= \ln|-x+2|^{-1} + c$$

$$= \ln\left(\frac{1}{-x+2}\right) + c.$$

$$\underline{\underline{\text{Ex}}} \quad \int \frac{1}{(x^2+1)} dx = \text{Trig Substitution.}$$

$$\underline{\underline{\text{Ex}}} \quad \int e^x dx = e^x + c$$

$$\underline{\underline{\text{Ex}}} \quad \int e^{2x+3} dx = \left(\frac{e^{2x+3}}{2} + c\right)$$

$$\frac{d}{dx} \left[\frac{e^{2x+3}}{2} + c \right] = \frac{1}{2} e^{2x+3} \cdot 2$$

$$= e^{2x+3}$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$$

Ex

$$\int 7 dy = 7y$$

$$x \leftrightarrow y$$

$$7 \int 1 dy$$

$$= 7y$$

$$y = a^2$$

$$\int x dy$$

$$\int 7 dx = 7x + c$$

$$\int y dx = yx + c$$

Ex

$$\int \frac{1}{\sqrt[3]{4x+7}} dx$$

$$= \int \frac{1}{(4x+7)^{1/3}} dx$$

$$= \int (4x+7)^{-1/3} dx$$

$$= \frac{(4x+7)^{-1/3+1}}{(-1/3+1) \cdot 4} + c = \frac{3(4x+7)^{2/3}}{2 \cdot 4} + c$$

$$= \frac{3(4x+7)^{2/3}}{8} + c$$

$$= \frac{\ln(4x+7)}{4} + c$$

Ex

$$\int \left(e^{5x} + \frac{8}{5x-3} \right) dx$$

$$= \frac{e^{5x}}{5} + \frac{8 \ln|5x-3|}{5} + c \quad \checkmark$$