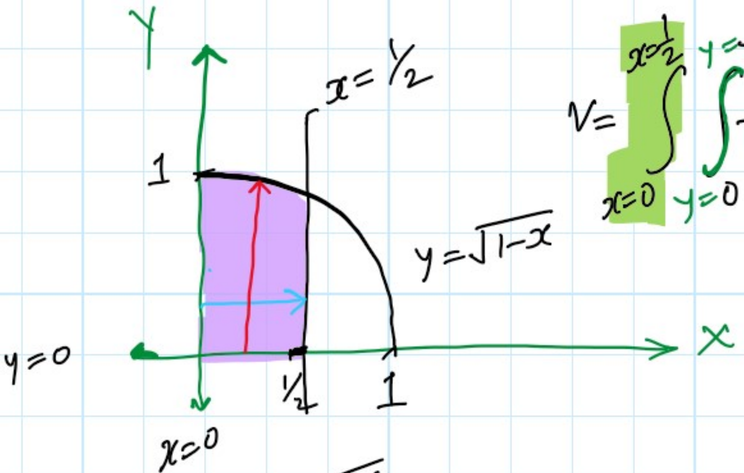
Double integral(Questions)

Wednesday, July 8, 2020 10:54 AM

Ex. $\displaystyle\iint_R 2xy \, dA$; $R: \{(x, y) \mid 0 \le x \le \tfrac{1}{2},\ 0 \le y \le \sqrt{1-x}\}$



$$V = \int_{x=0}^{x=\frac{1}{2}} \int_{y=0}^{y=\sqrt{1-x}} f(x,y)\,dy\,dx.$$

The outer integral must have constant limit.

$$V = \int_{x=0}^{x=\frac{1}{2}} \int_{y=0}^{y=\sqrt{1-x}} 2xy \; dy\,dx.$$

$$= \int_{x=0}^{x=\frac{1}{2}} \left[\frac{2xy^2}{2}\right]_0^{\sqrt{1-x}} dx \quad = \int_{x=0}^{x=\frac{1}{2}} x(1-x) - 0 \; dx.$$

$$= \int_{x=0}^{x=\frac{1}{2}} x - x^2 \; dx \quad = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1/2}$$

$$= \frac{1/4}{2} - \frac{1/8}{3} - (0)$$

$$= \frac{1}{8} - \frac{1}{24} = \frac{3-1}{24} = \frac{2}{24} = \frac{1}{12}$$

Fubini's theorem for general region:-

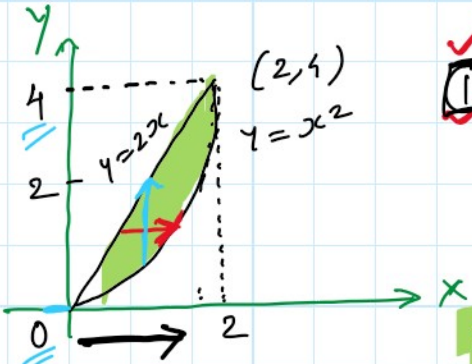
$$\int_a^b \int_{y=g_1(x)}^{y=g_1(x)} f(x,y)\,dy\,dx. = \int_c^d \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx\,dy$$

$$\int_{a}\int_{y=g_1(x)} f(x,y)\, dy\, dx = \int_{c}\int_{x=h_1(y)} f(x,y)\, dx\, dy$$

Ex: Region bound by $y = 2x$ & $y = x^2$

$f(x,y)$

Solⁿ



① $$\int_{y=0}^{y=4}\int_{x=y/2}^{x=\sqrt{y}} f(x,y)\, dx\, dy$$

② $$\int_{x=0}^{x=2}\int_{y=x^2}^{y=2x} f(x,y)\, dy\, dx.$$

$y = 2x. \Rightarrow x = \frac{y}{2}$

$y = x^2 \Rightarrow x = \sqrt{y}$

Ex: $$\iint_R \frac{x}{1+xy}\, dA \quad , \quad R : \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$$

Solⁿ

$$V = \int_0^1\int_0^1 \frac{x}{1+xy}\, dy\, dx.$$

$$I = \int \frac{f'(x)}{f(x)}\, dx.$$

$$= \int_0^1 \Big[\ln|1+xy|\Big]_0^1\, dx.$$

$$= \int_0^1 \ln|1+x| - \ln|1+0|\, dx.$$

$$= \int_0^1 \ln(1+x) - 0\, dx.$$

$$= \int_0^1 \ln(1+x)\, dx.$$

$\ln x.$

$d(x\ln x - x)$

$$\int_0$$

$$= \Big[(1+x)\ln(1+x) - (1+x) \Big]_{x=0}^{x=1}$$

$u = \ln(1+x)$, $v = x$

$du = \frac{1}{1+x}$, $dv = 1$

$$= (1+1)\ln(1+1) - (1+1) - \Big((1+0)\ln(1+0) - (1+0) \Big)$$

$uv - \int \frac{x}{1+x}\,dx.$

$\boxed{x\ln(1+x) - \ln(1+x)}$

$$= 2\ln 2 - 2 + 1$$

$$= 2\ln 2 - 1 = \ln 4 - 1$$

Ex. $\iint \frac{\ln y}{y}\,dA$, $R: \{(x,y) \mid 0 \le x \le \pi,\ e^{-2x} \le y \le e^{\cos x}\}$

Sol^n

$$V = \int_{x=0}^{x=\pi} \int_{y=e^{-2x}}^{y=e^{\cos x}} \frac{\ln y}{y}\,dy\,dx.$$

ANS. $\left(\frac{\pi}{4} - \frac{2}{3}\pi^3\right)$

$u_y = \ln y$

$du_y = \frac{1}{y}\,dy.$

$y = e^{-2x} \rightarrow u = \ln e^{-2x} = -2x$

$y = e^{\cos x} \rightarrow u = \cos x.$

$$= \int_{x=0}^{x=\pi} \int_{-2x}^{\cos x} u\,du\,dx.$$

$$= \int_{x=0}^{x=\pi} \left[\frac{u^2}{2}\right]_{-2x}^{\cos x} dx.$$

$\frac{1}{4}(1+\cos 2x) = \frac{2\cos^2 x}{4}$

$= \frac{1}{2}\cos^2 x.$

$$= \int_{x=0}^{x=\pi} \left(\frac{\cos^2 x}{2} - \frac{1}{2}\cdot 4x^2\right) dx$$

$$= \int_{x=0}^{x=\pi} \frac{1}{4}(1+\cos 2x) - 2x^2\,dx.$$

$^3\ ]^{\pi}$

$$\int_{x=0} \quad 4$$

$$= \left[\frac{1}{4}x + \frac{\sin 2x}{8} - \frac{2x^3}{3}\right]_0^{\pi}$$

$$= \frac{\pi}{4} + \frac{(\sin 2\pi)^{\to 0}}{8} - \frac{2\pi^3}{3} - (0+0-0)$$

$$= \frac{\pi}{4} - \frac{2\pi^3}{3}$$

Ex $\iint_R ye^{xy}\, dA \quad ; \quad R = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1\}$

$$V = \int_0^1 \int_0^1 ye^{xy}\, dx\, dy$$

ANS:- $(e-2)$

$u_x = xy$

$du_x = y\, dx.$

$x=0, \; u=0$

$x=1, \; u=y$

$$= \int_0^1 \int_0^y e^u\, du\, dy$$

$$= \int_0^1 [e^u]_0^y\, dy$$

$$= \int_0^1 e^y - e^0\, dy = \int_0^1 e^y - 1\, dy$$

$$= [e^y - y]_0^1 = e^1 - 1 - e^0 + 0$$

$$= e - 2$$