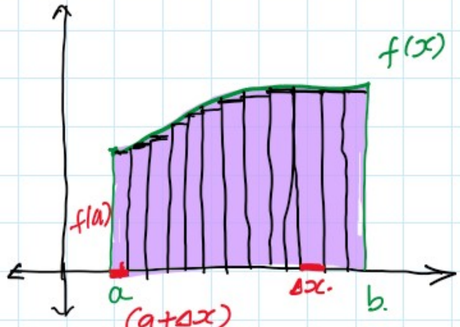
Double integral

Monday, July 6, 2020 11:02 AM

Riemann approximation:-

Calc-1.



$a \le x \le b$

$f(a+\Delta x)$

$= \frac{a+(a+\Delta x)}{2}$

$[a,b]$ Divide into 'n' equal parts.

$\frac{b-a}{n} = \Delta x$. (width of each rectangle)

→ Find 1 point on each Subintervals x_K^*

→ Find Height at each $x_k^* \Rightarrow f(x_k^*)$

→ Find area of each rectangle. $f(x_k^*)\,\Delta x$.

→ Add all the area.

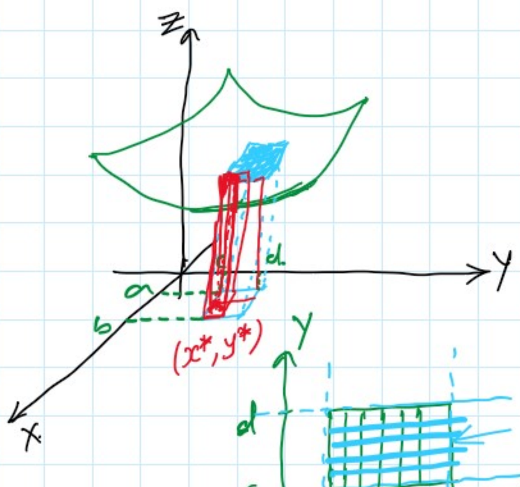
$$A = \sum_{K=1}^{n} f(x_K^*) \cdot \Delta x \rightarrow \text{Approximate}$$

$\Delta x \to 0$
$n \to \infty$

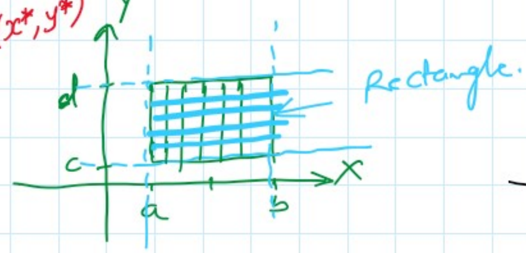
$$A = \lim_{n\to\infty} \sum_{K=1}^{n} f(x_K^*) \cdot \Delta x \longrightarrow \text{Exact area}$$

$$A = \int_a^b f(x_k^*) \cdot dx \longleftarrow \text{Exact area.}$$

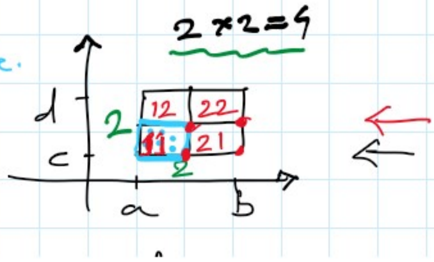
$z = f(x,y)$



$a \le x \le b$
$c \le y \le d$ } Region.



$2 \times 2 = 4$



Cut rectangle so that they are equal in number of x partition.

'm' → no of x partition.
'n' → no. of y-partition
} $m \times n$

$$\underset{(\text{width})}{\Delta x} = \frac{b-a}{m}, \quad \underset{(\text{length})}{\Delta y} = \frac{d-c}{n} \Rightarrow \text{Area} = \Delta x \cdot \Delta y$$

$z = f(x^*, y^*)$ ⟵ Height of cuboid.

→ Volume = Surface area × Height.

$= \Delta x \cdot \Delta y \times f(x^*_{ij}, y^*_{ij})$

(x^*_{12}, y^*_{12})

$$\to \text{Volume} = \sum_{i=1}^{m}\sum_{j=1}^{n} \Delta x \cdot \Delta y \cdot f(x^*_{ij}, y^*_{ij}) \quad \text{Approximate.}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} f(x^*_{ij}, y^*_{ij})\, \Delta x \cdot \Delta y$$

$$\to \text{volume} = \left(\lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n}\right) f(x^*_{ij}, y^*_{ij})\, \Delta x \cdot \Delta y$$

$\Delta A = \Delta x \cdot \Delta y = \Delta y \cdot \Delta x$.

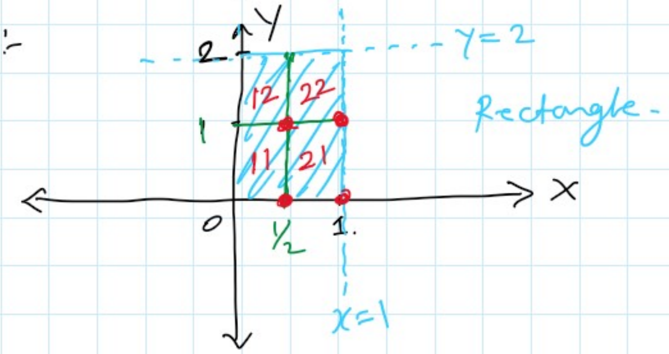
$$= \iint_{\text{Region}} f(x,y)\, dx \cdot dy$$

Ex: Find volume for $z = 8 - 2x^2 - y^2$ on

$R: \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 2\}$

For $m=2, n=2$, use lower right corner points.

Sol^n:-



$\Delta x = \frac{1-0}{2} = \frac{1}{2}$

$\Delta y = \frac{2-0}{2} = 1$

$\Delta x \cdot \Delta y = \frac{1}{2} \times 1 = \frac{1}{2}$

$[f(\ \ ) + f(\ \ ) + f(1,0) + f(1,1)] \times \frac{1}{2}$

$$V \approx \left[f\left(\tfrac{1}{2}, 0\right) + f\left(\tfrac{1}{2}, 1\right) + f(1,0) + f(1,1) \right] \times \frac{1}{2}$$

$$= \left[\frac{15}{2} + \frac{13}{2} + 6 + 5 \right] \times \frac{1}{2}$$

$$\boxed{V = \frac{25}{2}}$$

$$V = \iint_R f(x,y)\, dx\, dy = \iint_R f(x,y)\, dy\, dx.$$

For rectangular region : $R = \left\{ (x,y) \,\middle|\, \begin{matrix} a \le x \le b, \\ c \le y \le d \end{matrix} \right\}$

$$\int_{y=c}^{y=d} \int_{x=a}^{x=b} f(x,y)\, dx\, dy = \int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x,y)\, dy\, dx.$$

we will chose one which is easier to handle.

Ex:

$$\iint_R x + y^2 \, dA \quad ; \quad R : \left\{ (x,y) \,\middle|\, \begin{matrix} 0 \le x \le 1, \\ -1 \le y \le 2 \end{matrix} \right\}$$

Soln:

$$V = \int_{y=-1}^{y=2} \int_{x=0}^{x=1} x + y^2 \, dx\, dy$$

$$= \int_{y=-1}^{y=2} \left[\frac{x^2}{2} + xy^2 \right]_0^1 dy$$

$$= \int_{y=-1}^{y=2} \left(\frac{1}{2} + y^2 \right) - (0)\, dy$$

once integrated w.r.t. x, x should disappear completely

$$\left[\quad \frac{y^3}{} \quad \right]^2$$

$y=-1$ completely

$$= \left[\frac{1}{2}y + \frac{y^3}{3}\right]_{-1}^{2}$$

$$= 1 + \frac{8}{3} - \left(\frac{-1}{2} - \frac{1}{3}\right)$$

$$= \frac{11}{3} + \left(\frac{5}{6}\right) = \frac{27}{6} = \frac{9}{2}$$