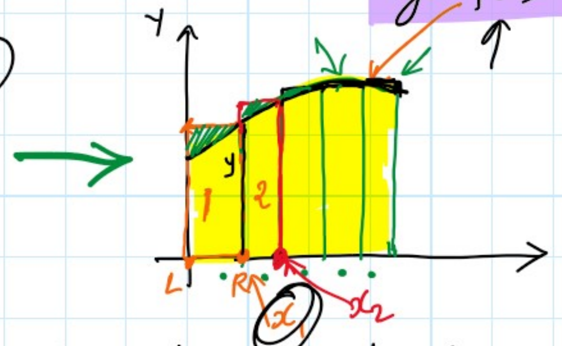
Integration(Antiderivative)

Thursday, January 28, 2021 5:51 AM

① $y = f(x)$



5 rectangles.

width $= \Delta x$.

length $= f(x_1)$

How to find Area.??

Ans → Exact

Riemann Sum approximation:-

② Antiderivative:-

$y=f(x)$

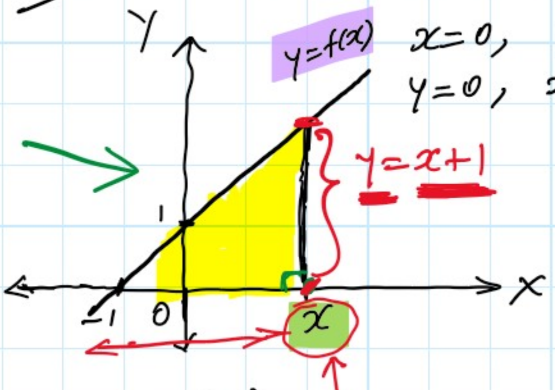
If $A(x)$ is the area function for some $f(x)$ on $[a,b]$

Then, $A'(x) = f(x)$ (*)

→ this implies to find the area under $f(x)$ we must 'undo' $A'(x)$ to find $A(x)$

ANTIDERIVATIVE

Ex: $y = f(x) = x+1$ Find area on $[-1, x]$



$x=0, \; y=1$

$y=0, \; x=-1$

$x-(-1)$
$= x+1$

Base $= x+1$

height $= x+1$

$A(x) = \frac{1}{2}\, b \times h$

$A(x) = \frac{1}{2} \times (x+1)(x+1)$

$A(x) = \frac{1}{2}\,(x^2+2x+1)$

$A'(x) = \frac{1}{2}\,[2x+2 \quad]$

$\boxed{A'(x) = f(x)}$

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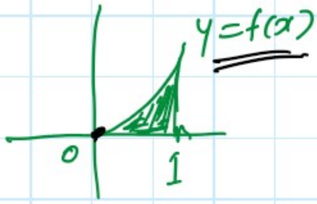
$A'(x) = \frac{1}{2}[2x + 2]$

$\Rightarrow A'(x) = x + 1 \leftarrow f(x)$

Ex find Area under $f(x) = x^2$ in $[0,1]$



$A'(x) = f(x)$

$\Rightarrow A'(x) = x^2$

Antiderivative

$A(x) =$

? Can you find a function [A(x)] whose derivative is x^2?

$A(x) = x^3$, $A'(x) = 3x^2$,

$\Rightarrow A(x) = \frac{x^3}{3}$, $A'(x) = x^2$

$\Rightarrow A(x) = \frac{x^3}{3} + 5$, $A'(x) = x^2$

$\Rightarrow A(x) = \frac{x^3}{3} - 7$, $A'(x) = x^2$

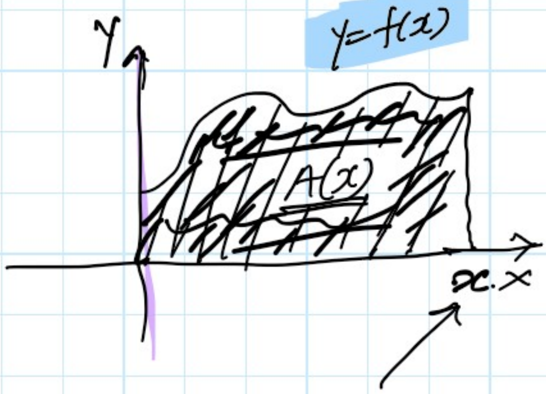
$A(x) = \frac{x^3}{3} + C$, $C \in \mathbb{R}$.

$A'(x) = x^2 \leftarrow f(x)$.

$= \frac{x^{2+1}}{2+1}$

$= \frac{x^3}{3} + C$

family of curve.



$A'(x) = f(x)$

$A(x) = \text{Anti}(f(x))$

$A(x) = \int f(x) \cdot dx$.

Integration.

$$A'(x) = x^3 = f(x)$$

$$A(x) = \frac{x^4}{4} + c \Leftarrow A'(x) = \frac{4x^3}{4} + 0$$

$$= x^3 + 0$$

$$= x^3$$

$$\boxed{\begin{aligned} A'(x) &= f(x) = x^n, \quad n \neq -1 \\ A(x) &= \int f(x)\,dx = \frac{x^{n+1}}{n+1} + C \end{aligned}}$$

$C \in \mathbb{R}$

Ex Find the antiderivatives of each fn.

(a) $f(x) = \dfrac{1}{x^3}$

$A(x) =$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$$

$$= \frac{x^{-3+1}}{-3+1} + C, \quad C \in \mathbb{R}.$$

$$= \frac{x^{-2}}{-2} + C$$

$$= \frac{1}{-2x^2} + C$$

(b) $f(x) = \sqrt[5]{x^3}$

$$\int \sqrt[5]{x^3}\,dx = \frac{5}{8} x^{8/5} + C$$

Ex $I = \int (x^2 + \sqrt{x})\,dx$

$$= \int x^2\,dx + \int \sqrt{x}\,dx.$$

$$= \frac{x^3}{3} + \frac{2}{3}x^{3/2} + C$$

$$\left\{ \begin{aligned} &\frac{d}{dx}(u+v) \\ &\frac{du}{dx} + \frac{dv}{dx}. \end{aligned} \right\}$$

$$= \frac{x^3}{3} + \frac{\ }{3}x \quad + c$$

Indefinite Integration :-

$$\int_a^b f(x)\,dx = \text{finite value.}$$

Definite Integration

$$\frac{d}{dx}(x^2) = 2x$$

$$\int 2x\,dx = x^2$$

$$\frac{d}{dx} f(x) = g(x)$$

$$\int g(x)\,dx = f(x)$$

$$\frac{d}{dx} e^x = e^x$$

$$\int e^x\,dx = e^x + c.$$

Ex (a) $\int \frac{3}{x}\,dx$

$$\cancel{I = \int 3x^{-1}\,dx}$$

$$\cancel{= 3\,\frac{x^{-1+1}}{-1+1}}$$

(b) $\int \frac{e^x}{5}\,dx.$

$$= \frac{1}{5}\int e^x\,dx.$$

$$= \frac{1}{5}\cdot e^x + c.$$

$c \in \mathbb{R}.$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

$$\int \frac{1}{x}\,dx = \ln x$$

$$\int \frac{3}{x}\,dx = 3\int \frac{1}{x}\,dx$$

$$\int \frac{3}{x}\,dx = 3\int \frac{1}{x}\,dx$$

$$= \underline{\underline{3\ln x.}} + c.$$

Find the indefinite integral.

1 $\int \frac{6}{x}\,dx$

2 $\int 5e^{u}\,du$

3 $\int \frac{1}{2x}\,dx$

4 $\int \frac{e^{x}}{3}\,dx$

5 $\int (3x+2)^2\,dx$

6 $\int \ln(e^{x+1})\,dx$

7 $\int (t^2(t+3))\,dt$

8 $\int e^{\ln(3x)}\,dx$

9 $\int \frac{x^4+3x^2+2x}{x}\,dx$

10 $\int \frac{e^{u}-4}{2}\,du$