Test

Wednesday, July 8, 2020 6:03 PM

Comparison test for convergence:-

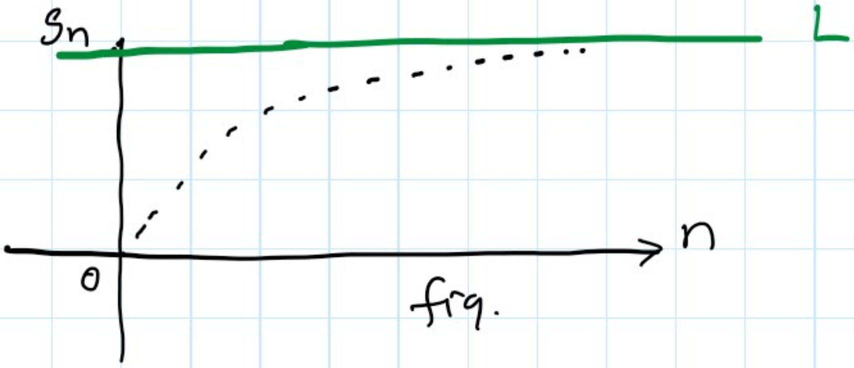

fig.

Definition:- A sequence $\{a_n\}$ is monotonic if its terms are non-decreasing,

i.e. $a_1 \le a_2 \le a_3 \le --- \le a_n ---$

or if it terms are non-increasing

i.e. $a_1 \ge a_2 \ge a_3 \ge ---- \ge a_n ---$.

Thm: If a sequence is both bounded and monotonic then it converges.

fig:- monotonic increasing sequence.
& it is bounded above. Hence converges

fig 1. monotonic increasing sequence.
& it is bounded above, Hence converges.

Thm: $\sum a_n$ & $\sum b_n$ $\quad 0 \le a_n \le b_n$
for all n. Then.

Contrapositive.

1) If $\sum b_n$ converges. then $\sum a_n$ converges

2) If $\sum a_n$ diverges then $\sum b_n$ diverges.

Limitation
- If $\sum b_n$ diverges we can not say about $\sum a_n$
- If $\sum a_n$ converges then also no result for $\sum b_n$.

Thm!:

$$\frac{1}{n^p} \ge \frac{1}{n} \quad \text{for } 0 < p \le 1$$

$$\frac{1}{n} \le \frac{1}{n^p} \qquad a_n \le b_n$$

2) $\frac{1}{n}$ diverges $\therefore$ $\frac{1}{n^p}$ diverges.

Ex: Converges/diverges.

a) $\sum \frac{\sin^2 n}{3^n}$

$0 \le \sin^2 n \le 1.$

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$/3^n$ $\quad 0 \leq \frac{\sin^2 n}{3^n} \leq \frac{1}{3^n}$ } sequence.

$b_n = \frac{1}{3^n}$ — G.P. $\left(r = \frac{1}{3} < 1\right)$

$\frac{1}{3^n}$ converges $\Rightarrow$ $\frac{\sin^2 n}{3^n}$ also converges.

b) $\sum \frac{n}{3n+1}.$

$\frac{1}{n} < \frac{n}{3n+1}.$ $\quad$ $n > 4$

$\frac{1}{3} \geq \frac{n}{3n+1}$

$\frac{n}{3n+1} \leq \frac{1}{3}$

$n=1 \quad \frac{1}{3} > \frac{1}{4}$

$\sum_{n=1}^{\infty} \frac{1}{3}$ = divergent.

$n=2 \quad \frac{1}{3} > \frac{\cancel{2}}{\cancel{7}\,3}$

$n=5, \quad \frac{1}{5} < \frac{5}{16}$

$\frac{1}{5} < \frac{1}{3.2}$

$n=10 \quad \frac{1}{3} > \frac{10}{31}$

$\frac{1}{3} > \frac{1}{3.1}$

$n=10, \quad \frac{1}{10} < \frac{1}{3.1}$

$n=1000, \quad \frac{1}{3} \quad \frac{1}{3.001}$

$a_n = \frac{1}{n}$ (divergent series)

$a_n = \frac{1}{n} < \frac{n}{3n+1} = b_n$

$\therefore \quad b_n = \frac{n}{3n+1}$ is also divergent.

Ex Establish whether or not the series

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$$\sum_{K=1}^{\infty} \frac{1}{2^K+3} \text{ converges.}$$

$$\frac{1}{2^K+3} < \left(\frac{1}{2^K}\right)$$

$\therefore \downarrow$ converges $\qquad \therefore \downarrow$ converges

Limit comparison test for convergence:-

Thm:- Let $a_n > 0$ & $b_n > 0$. for all $n \geq N$

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ (positive real no)

then both Σa_n and Σb_n converges or both diverges.

General Harmonic series: $a_n = \frac{1}{an+b}$

$$\Sigma \frac{1}{an+b} \qquad b_n = \frac{1}{n} \text{ (Harmonic series)}$$

$$\lim_{n\to\infty} \frac{\left(\frac{1}{an+b}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{n}{an+b} = \frac{1}{a} > 0$$

$\because \frac{1}{n}$ is diverges $\therefore \frac{1}{an+b}$ also diverges.

Ex: Show that the series $\sum_{k=1}^{\infty}\left(\frac{1}{2^k-1}\right)$ is Convergent.

Solⁿ:

$$\lim_{k\to\infty}\frac{\frac{1}{2^k-1}}{\left(\frac{1}{2^k}\right)} = \lim_{k\to\infty}\frac{2^k}{2^k-1}$$

$$= \lim_{k\to\infty}\frac{2^k/2^k}{\frac{2^k}{2^k}-\frac{1}{2^k}} =$$

$$= \lim_{k\to\infty}\frac{1}{1-\left(\frac{1}{2^k}\right)} = 1. > 0$$

$\frac{1}{2^k}$ is G.S with ratio $\frac{1}{2} < 1$

$\Longrightarrow$ converges.

$\therefore \frac{1}{2^k-1} \Longrightarrow$ converges.

Ex: By LCT.

a) $\sum \frac{n}{n^2+1}$.

Idea of selecting p-series test ??

$$\sum \frac{n}{n^2} = \sum \frac{1}{n} \longleftarrow \text{reference series.}$$

$\frac{n}{}$

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = 1. > 0$$

$\frac{1}{n} \to$ divergent

$\frac{n}{n^2+1} \longrightarrow$ divergent.

Ex: $\sum \frac{2\sqrt{n}-3}{3n^2+4\sqrt{n}}$

Compare to $\sum \frac{\sqrt{n}}{n^2} = \sum \frac{1}{n^{3/2}}$

$$\lim_{n\to\infty} \frac{\frac{2\sqrt{n}-3}{3n^2+4\sqrt{n}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{(2\sqrt{n}-3)\,n^{3/2}}{3n^2+4\sqrt{n}} = \frac{2}{3}$$

$\frac{1}{n^{3/2}} \longrightarrow$ converges. $p > 1$.

Given series $\longrightarrow$ converges by LCT

Ratio Test for convergence:-

Now we don't require reference series.

Ratio test: Let $a_n > 0$ for $n \geq 1$

$\lim \frac{a_{n+1}}{\ldots} \;\; 1$. Then $\sum^{\infty} a_n$ converges

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L. \text{ Then } \sum_{n=1}^{\infty} a_n \text{ converges}$$

if $L < 1$,

and diverges if $L > 1$.

when $L = 1$. (Test is inconclusive)