Central limit thm

Friday, February 19, 2021 4:00 PM

1-c) $P(Z=z)=\frac{5-z}{10}$, $z=1,2,3,4$

z	1	2	3	4
$P(Z=z)$	$4/10$	$3/10$	$2/10$	$1/10$

$E(\bar{Z}_{10})=?$ $\quad var(\bar{Z}_{10})=?$

$E(\bar{Z}_{10})=E(Z)$
$=2$

$var(\bar{Z}_{10})=0.1$

③ a) $E(X)=5$
$var(X)=8.5$

b) $E(\bar{X}_{12})=5$
$var(\bar{X}_{12})=\frac{8.5}{12}=0.708$

\# n observation. , $X \sim N(\mu, \sigma^2)$

$\Rightarrow \bar{x}=\frac{x_1+x_2+x_3+\dots+x_n}{n}$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Q Packets of cereal are labelled as containing 500 grams of cereal. The actual contents can be modelled by a **normal distribution** with mean 503 g and standard deviation 7 g.

What is the probability that the mean contents of a randomly selected sample of 10 packets will be under 498 g?

ANS

$n = 10$, $X \sim N(503, 7^2)$

$\bar{X} \sim N\left(503, \frac{7^2}{10}\right)$

$P(\bar{X} < 498) = \phi(-2.259)$ $\qquad Z_{score} = \frac{x - \mu}{\sigma}$

$= 1 - \phi(2.259)$

$= 1 - 0.9881$

$= 0.0119$

The volume of liquid dispensed into bottles is normally distributed with mean 500 ml and standard deviation 6.2 ml.

i) A random sample of 5 bottles is taken. What is the probability that the mean volume in the 5 bottles is at least 504 ml?

ii) Another random sample is to be taken. What is the minimum sample size needed for there to be no more than a 1% probability that the mean volume is greater than 501 ml?

$P(\bar{X}_n > 501) \leq 0.01$

$\Rightarrow P(\bar{X}_n < 501) \geq 0.99$

$\left(\frac{501 - 500}{\ldots}\right) \geq 0.99$

$$\phi\left(\frac{501-500}{\frac{6.2}{\sqrt{n}}}\right) \geq \underline{\underline{0.99}}$$

$$\left\{\frac{501-500}{\frac{6.2}{\sqrt{n}}} \geq 2.326\right\}$$

$$\sqrt{n} \geq 2.326 \times 6.2 = 14.42$$

$$n \geq (14.42)^2 = 207.91$$

$$\boxed{n \geq 207.91}$$

2. Packets of biscuits are labelled as containing 350 grams. The actual contents can be modelled by a normal distribution with mean 352 grams and standard deviation 4.5 grams.

a) Find the probability that the mean contents of a random sample of 10 packets is at least 350 grams.

b) What is the smallest size of random sample for which there is probability of under 1% that the mean contents is under 350 grams?

(a) $$P(\bar{X}_{10} \geq 350) = P\left(Z > \frac{350-352}{\frac{4.5}{\sqrt{10}}}\right)$$

$$= 0.920$$

$$Z = \frac{x-\mu}{\sigma}$$

(b) $$P(\bar{X}_{10} < 350) < \underline{0.01}$$

$$\frac{350-352}{\frac{4.5}{\sqrt{n}}} < -2.326$$

$$\frac{4.5}{\sqrt{n}}$$

$$\Rightarrow -\frac{2\sqrt{n}}{4.5} < -2.326$$

$$\Rightarrow \cancel{4.5}\frac{(-2\sqrt{n})}{\cancel{4.5}} < -2.326 \times 4.5$$

$$\Rightarrow \sqrt{n} > \frac{2.326 \times 4.5}{2}$$

$$\Rightarrow n > 27.39$$

$$\Rightarrow \boxed{n = 28}$$

CLT! (central limit theorem).

$X \sim$ (don't know the distribution)

$\bar{X}$ whether $\bar{X}$ follows normal distribution or Not ?

If sample size ≥ 30

then $\bar{X}$ follows the normal distribution.

On a flight there are 43 bags checked in. Historical records show that on that route the mean weight of checked bags is 12.1 kg with a standard deviation of 3.8 kg.

Normal.

What is the probability that the mean contents of the checked bags on this flight will be under 11 kg? (You may treat the bags on this flight as though they are a random sample of bags checked in on that route.)

Sample size (43) $\geq$ 30.

variance.

. mean of sample$(\bar{x}) \sim N\left(12.1, \frac{3.8^2}{43}\right)$

$P(\bar{x} < 11) = 0.0288 = 2.9\%$