Curve sketching

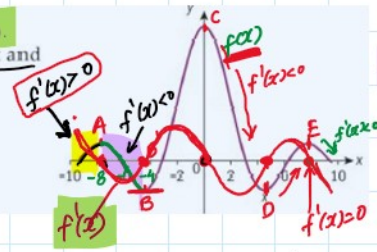
Thursday, January 14, 2021 5:58 AM

$f(x)$, $f'(x)$, $f''(x)$

1) turning points / POI ← $\frac{dy}{dx} = 0$. (stationary point)
2) Asymptotes.
3) x- intercepts.

Ex

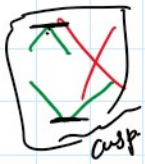
The following diagram shows the graph of $y = f(x)$.
Copy the graph, and sketch the graphs of the first and second derivatives of f on the same set of axes.



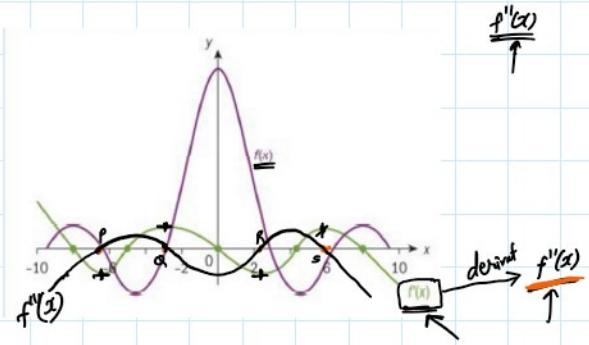
$f'(x)$, $f''(x)$

$f'(x) > 0$ (above x-axis)
$f'(x) < 0$ (below x-axis)
* Graph should be smooth.

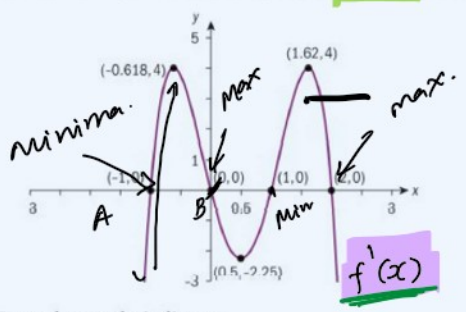
$f'(x) = 0$, at stationary points.
(Gradient) (tangent drawn // x axis)

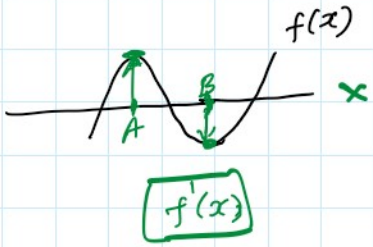


$f'(x) > 0$ — Graph is increasing
$f'(x) < 0$ — Graph is decreasing.



The follow diagram shows the graph of $y = f'(x)$ for a function f.



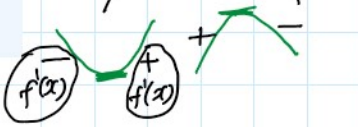


stationary

From the graph, indicate:

a the x-coordinate of any points where f has turning points and determine the nature of these points $x = -1, 0, 1, 2$
b the intervals where f is i increasing and ii decreasing $f'(x) > 0$, $f'(x) < 0$
c the intervals where f is i concave up and ii concave down.
d Sketch a possible graph of f using your answers to parts **a**, **b** and **c**.

minima or maxima.



decreasing $x < -1$; $0 < x < 1$; $x > 2$

decreasing $x<-1$; $0<x<1$; $x>2$

increasing $-1<x<0$, $1<x<2$

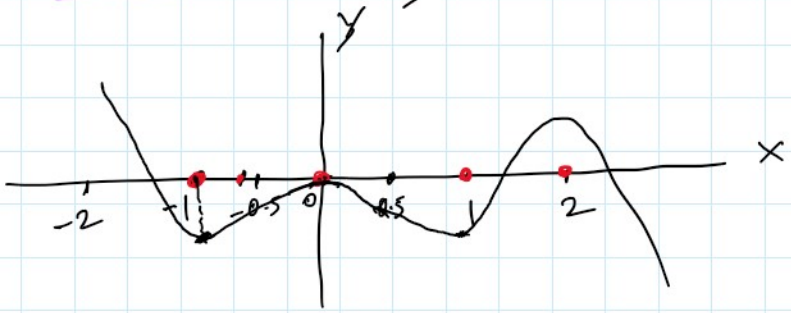
$f''(x) \geq 0$ concave up. — $f'(x)$ – increasing

$f''(x) \leq 0$ concave down. — $f'(x)$ – decreasing.

Concave up: $x<-0.618$, $0.5<x<1.62$

concave down :- $-0.618<x<0.5$, $x>1.62$

(d)

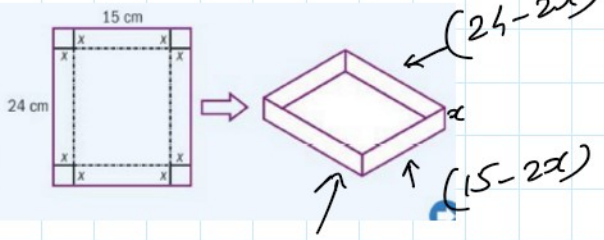


Application of Derivative.

(optimization).

Ex

A cardboard box manufacturer makes open boxes by cutting equal squares of side length x cm from the corners of a rectangular piece of cardboard measuring 15 cm by 24 cm. The sides are then folded up, as shown in the diagram. Find x so that the volume of the box is maximized, and find the maximum volume of the box. Check your answers graphically.



working of domain.

$$\begin{cases} 24-2x>0 \\ 15-2x>0 \end{cases}$$

$24-2x>0$

$24>2x$

$\Rightarrow x<12$

$15-2x>0$

$15>2x$

$7.5>x$

$x<7.5$

$0<x<7.5$

$V = (15-2x)(24-2x)x$.

$V = f(x) = 360x - 78x^2 + 4x^3$

$f'(x) = 0$

$360 - 156x + 12x^2 = 0$.

$12(30 - 13x + x^2) = 0$

$x^2 - 13x + 30 = 0$

$(x-10)(x-3) = 0$.

$x = 10$, $x = 3$ ← stationary points.

↑ — V. IMP

Check domain. — V. IMP

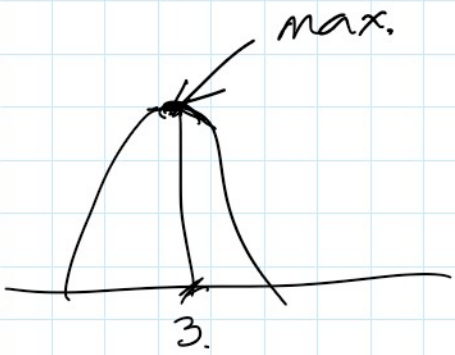
$f''(x) = -156 + 24x$.

$x = 3$ (max or min).

$\frac{d^2y}{dx^2} < 0$ — (max).

$f''(3) = -156 + 24(3) < 0$

V is maximum at $x = 3$.



$V = f(3) = (15-6)(24-6)3$

$= (9)(18)(3)$

$= \underline{486}$