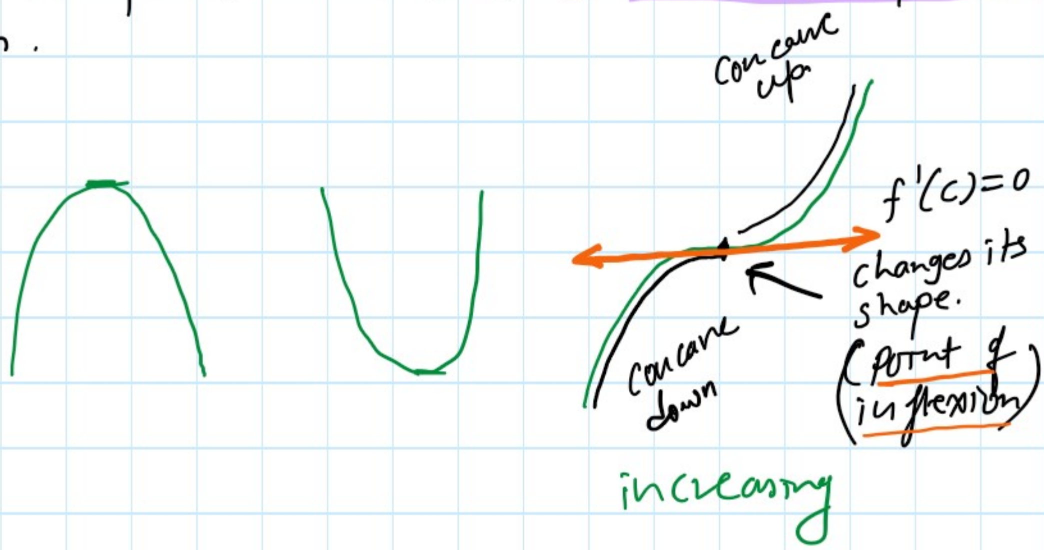
Point of inflection

Thursday, January 7, 2021 5:50 AM

If f has a point of inflexion at $x=c$ and $f'(c)=0$, the point is called a **horizontal point of inflexion**.



Ex

Find and classify all points of inflexion of $f(x) = 3x^4 + 4x^3 - 2$.

$f'(x) = 12x^3 + 12x^2$

$f''(x) = 36x^2 + 24x.$

set $f''(x) = 0$

$36x^2 + 24x = 0$

$\Rightarrow x^1(36x+24)^1 = 0$

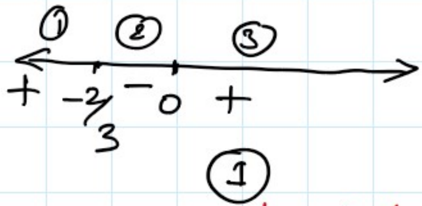
$x = 0$ or $36x + 24 = 0$

$x = 0$ $\Rightarrow x = -\frac{24}{36} = \frac{-2}{3}$

1) $f''(x) > 0$ concave up.

2) $f''(x) < 0$ concave down.

x	$x < -\frac{2}{3}$	$-\frac{2}{3} < x < 0$	$x > 0$
$f''(x)$	+	−	+



power has to be odd.

$x = -\frac{2}{3}$ $x = 0$

Concave — Concave

concave

3 ——

concave up | concave down | concave up.

odd.

$x = -\frac{2}{3}$, 0 ← point of inflexion.

$f'(c) = 0$ ← To be horizontal point of inflexion (POI).

$f'(x) = 12x^3 + 12x^2$

$f'(0) = 0$, $f'(-\frac{2}{3}) \neq 0$

Horizontal point of inflexion | point of inflexion.

Summary: $f''(x) = 0$ ← point of inflexion.

↓

$f'(x) = 0$ ← Horizontal point of inflexion.

* HW – Check it by graph.

Ex

Find and classify any turning points and points of inflexion of $y = (x + 1)(x - 3)^3$ and justify your answers. Confirm your answers graphically.

$y = (x+1)(x-3)^3$

$\frac{dy}{dx} = 4x(x-3)^2$ — 1

$\boxed{x = 0}$ and $\boxed{x = 3}$ ← classify

$\frac{d^2y}{dx^2} = 12(x-3)(x-1)$

$\frac{dy}{dx} = (x+1) \times 3(x-3)^2 + (x-3)^3$

$= (x-3)^2 [3x + 3 + x - 3]$

$= (x-3)^2 [4x]$

$\frac{d^2y}{dx^2} = 4x \cdot 2(x-3) + (x-3)^2 \cdot 4$

$$\frac{d^2y}{dx^2} = 12(x-3)(x-1)$$

$$\boxed{\left.\frac{d^2y}{dx^2}\right|_{x=0} = 36 > 0}$$ Minima

$$\boxed{\left.\frac{d^2y}{dx^2}\right|_{x=3} = 0}$$ ← Test is inclusive

$$\frac{d^2y}{dx^2} = 4x \cdot 2(x-3) + (x-3)^2 \cdot 4$$

$$= 4(x-3)[2x + x - 3]$$

$$= 4(x-3)[3x-3]$$

$$(*) \quad = 12(x-3)(x-1)$$

C.P. $\begin{cases} f''(x) > 0 & \text{— Minima} \\ f''(x) < 0 & \text{— Maxima.} \end{cases}$

$f''(x) = 0$ — Test is inclusive

$f''(x) = 0$ ← POI

$$\frac{d^2y}{dx^2} = 0$$

$$12(x-3)(x-1) = 0$$

$x = 1$ & $x = 3$

$x = 2.9$, $x = 3.1$

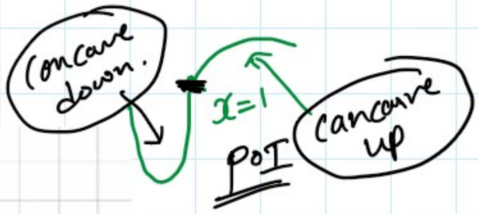
$$\boxed{\frac{d^2y}{dx^2} = 12(x-3)(x-1)}$$

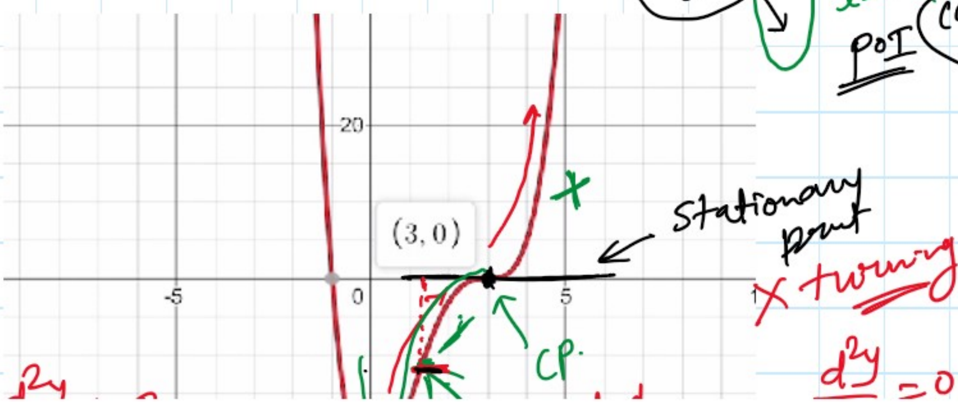
$\frac{d^2y}{dx^2} < 0$, $\frac{d^2y}{dx^2} > 0$ ← Concave up.

-ve +ve

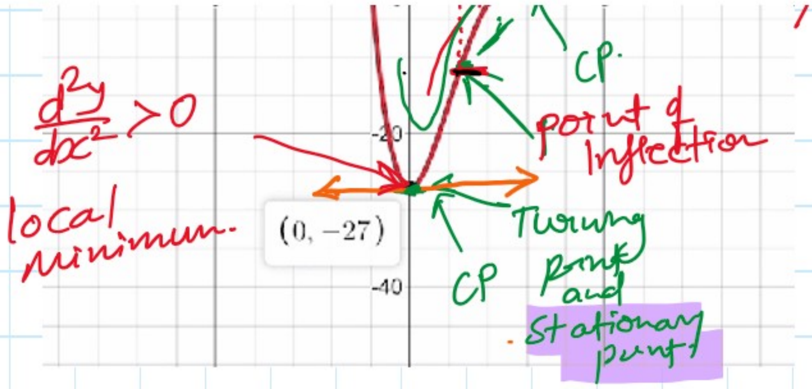
Concave down

Changes it concavity at $x = 3$. ← Stationary





$\frac{d^2y}{dx^2} = 0$



$\frac{d^2y}{dx^2} = 0$

Ex

$f(x) = 2x^3 + x^4$

a Find all turning points and points of inflexion; determine their nature and justify your answers.

b Find the intervals where the function is i concave up and ii concave down.

c Sketch the function, indicating any maxima, minima and points of inflexion.

ⓐ $f'(x) = 6x^2 + 4x^3$ ← $x=-1$, $x=1$

$x = 0$, $x = -\frac{3}{2}$

	$x<0$	$x>0$
$f'(x)$	+	+

$f''(x) = 12x + 12x^2$

$f''(0) = 0$ ← Test is inclusive

$f''(-\frac{3}{2}) = 9 > 0$ ← $x = -\frac{3}{2}$ (Minima).

ⓑ **POI** (curve change its shape)

$f''(x) = 0 \Rightarrow 12x^1(1+x)^1 = 0$

$x = 0, \; x = -1$

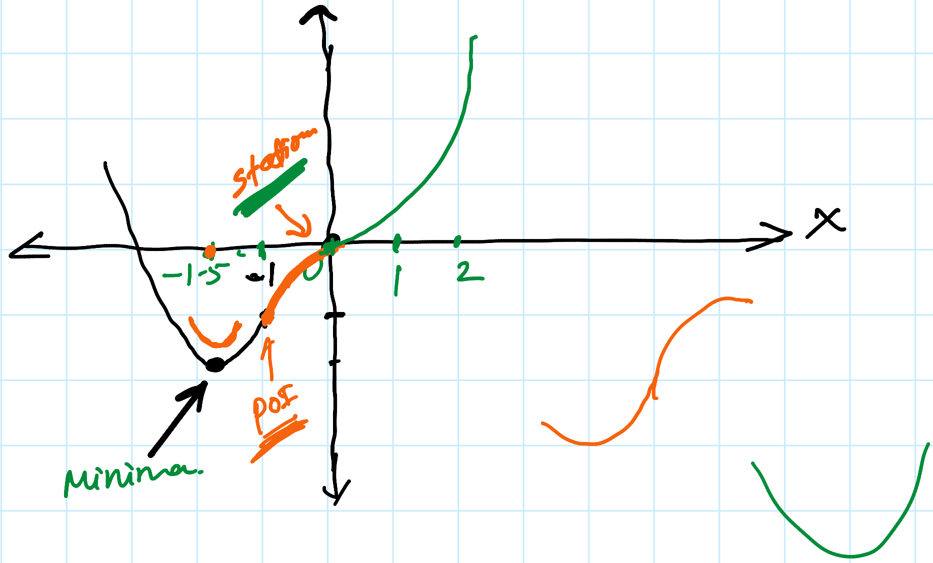
	$x<-1$	$-1<x<0$	$x>0$
sign $f''(x)$	+	−	+
Cancavity	up	down	up.

f is concave up. $(-\infty, -1)$ and $(0, \infty)$

f is cancave down on $(-1, 0)$

f is concave down on $(-1, 0)$

(c)



$$f(x) = 2x^3 + x^4$$

$$f\left(\frac{-3}{2}\right) = -2 \times \frac{27}{8} + \frac{81}{16} = -1.69$$