Quotient rule

Thursday, December 17, 2020 5:50 AM

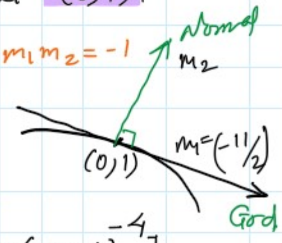
product rule:

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

Ex Find the eq^n of the normal to the curve

$$y = \frac{\sqrt{x+1}}{(2x+1)^3}, \quad x \neq -\frac{1}{2} \text{ at } (0,1).$$

$m_1 m_2 = -1$



Sol^n:

$$y = \sqrt{x+1}\,(2x+1)^{-3}$$

$$\Rightarrow y = (x+1)^{1/2}(2x+1)^{-3}$$

$$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-1/2}(2x+1)^{-3} + (x+1)^{1/2}\left[-6(2x+1)^{-4}\right]$$

at $\underrightarrow{x=0}$; $\frac{dy}{dx} = \frac{1}{2}(1)^{-1/2}(1)^{-3} + (1)^{1/2}\left[-6(1)^{-4}\right]$

$$= \frac{1}{2} - 6 = \frac{-11}{2}$$

$m_2 = \frac{-1}{m_1} = \frac{2}{11}$

Gradient of normal $\Rightarrow \frac{2}{11}$

Eq^n of Normal ? $\quad \boxed{y = \frac{2}{11}x + 1.}$

Ex

$$y = \sqrt{x+1}\,(3-x)^2$$

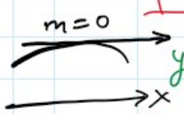
a) Show that $y' = \frac{(x-3)(5x+1)}{2\sqrt{x+1}}$

b) Find x-coordinates of all points on the graph of y where tangent to the curve is parallel to x-axis.

Sol^n:

$$\boxed{\cancel{y' = \frac{\frac{1}{2}(x+1)^{-1/2}(3-x)}{2\sqrt{x+1}}}}$$

$\frac{d}{dx}(-x)$
$-1x^{1-1}$
$-1x^0 = -1$



$$y = (x+1)^{1/2}(3-x)^2$$

$$y' = (x+1)^{1/2}\cdot 2(3-x)^{2-1}\cdot(0-1) + (3-x)^2\cdot\frac{1}{2}(x+1)^{-1/2}(1+0)$$

$$= -2(x+1)^{1/2}\cdot(3-x) + \frac{1}{2}\frac{(3-x)^2}{(x+1)^{1/2}}$$

$$= \frac{-4(x+1)(3-x) + (3-x)^2}{2(x+1)^{1/2}}.$$

$$= \frac{-4\left[3x - x^2 + 3 - x\right] + 9 - 6x + x^2}{2(x+1)^{1/2}}$$

$$= \frac{-12x + 4x^2 - 12 + 4x + 9 - 6x + x^2}{2(x+1)^{1/2}}$$

$$= \frac{5x^2 - 14x - 3}{2(x+1)^{1/2}} = \frac{5x^2 - 15x + x - 3}{2(x+1)^{1/2}}$$

$$\frac{dy}{dx} = y' = \frac{(x-3)(5x+1)}{2\sqrt{x+1}} \quad //$$

$$// \, x \text{axis} \Rightarrow \frac{dy}{dx} = 0 \Rightarrow \left[\frac{(x-3)(5x+1)}{2\sqrt{x+1}}\right] = 0.$$

$$\boxed{x \neq -1} \Rightarrow (x-3)(5x+1) = 0$$

$$x - 3 = 0 \quad \text{or} \quad 5x + 1 = 0$$

$$x = 3 \quad \text{or} \quad x = -1/5$$

Ex $\Rightarrow$ $y = \frac{(x^3-3)}{(3x-x^2)}$, $y' = ?$

Quotient rule.

$$y = \frac{u}{v}, \; v \neq 0, \quad y' = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$$u = x^3 - 3, \quad v = 3x - x^2$$

$$u' = 3x^2, \quad v' = 3 - 2x.$$

$$y' = \frac{(3x-x^2)(3x^2) - (x^3-3)(3-2x)}{(3x-x^2)^2}$$

$$y' = \frac{3x^2 - 6x + 9}{(3x-x^2)^2}$$

$$\frac{x^{-1/2} - \frac{1}{2}x^{1/2} + x^{1/2}}{\frac{1}{2}x^{1/2}}$$

No issue

Ex a) $y = \frac{\sqrt{x}}{2-x}$, y'

$$\left\{ \cancel{y' = \frac{x^{-1/2} - \frac{3}{2}x^{1/2}}{(2-x)^2}} \right\}$$

$$\frac{\frac{2}{2\sqrt{x}} + \frac{x}{2\sqrt{x}}}{(2-x)^2} = \frac{x^{-1/2} + \frac{1}{2}x^{1/2}}{(2-x)^2}$$

$$y' = \frac{(2-x) \cdot \frac{1}{2}x^{-1/2} - \sqrt{x}(-1)}{(2-x)^2}$$

$$= \frac{\frac{1}{2}\frac{(2-x)}{\sqrt{x}} \cancel{-}+ \frac{\sqrt{x}}{1}}{(2-x)^2} = \frac{\frac{(2-x) + (\sqrt{x})^2 \cdot 2}{2\sqrt{x}}}{(2-x)^2} = \frac{2 - x + 2x}{2\sqrt{x}(2-x)^2} = \frac{2+x}{2\sqrt{x}(2-x)^2} \; //$$

Ex Find the eq^ns of tangent and the normal to the curve $f(x) = \frac{1+\sqrt{x}}{x-1}$, $x \neq 1$.

at the point $(9, \frac{1}{2})$.

Sol^n

$$y = 24x - \frac{431}{2}$$

$$f'(x) = \frac{(x-1)\frac{1}{2}x^{-1/2} - (1+\sqrt{x})(1)}{(x-1)^2}; \; (x \neq 1)$$

$$f'(x) = \frac{(x-1)\frac{1}{2}x^{-1/2} - (1+\sqrt{x})(1)}{(x-1)^2} ; (x \neq 1)$$

$$\text{at } \underline{x=9} \rightarrow f'(9) = \frac{(9-1)\frac{1}{2}\cdot 9^{-1/2} - (1+\sqrt{9})}{(9-1)^2}$$

$$= \frac{\frac{4}{3} - 4}{8^2} = \frac{-8}{3\cdot 8^2} = \frac{-1}{24}$$

Gradient of tangent $= -1/24$

Gradient of Normal $= 24$

$\underline{\text{eqn of tangent}}$ at $(9, 1/2)$ $\Rightarrow$ $y - \frac{1}{2} = \frac{-1}{24}(x-9)$

$$y = -\frac{1}{24}x + \frac{9}{24} + \frac{1}{2}$$

$$\Rightarrow y = -\frac{1}{24}x + \frac{21}{24}$$

$\underline{\text{eqn of Normal}}$ at $(9, 1/2)$ $:\Rightarrow$ $\boxed{y = 24x - \frac{431}{2}}$