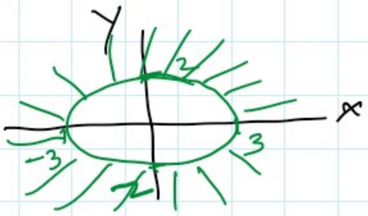
Revision Part 4

Saturday, July 4, 2020 10:52 AM

1) $f(x,y)=\sqrt{4x^2+9y^2-36}$, $g(x,y,z)=\dfrac{xyz}{\sqrt{x^2+y^2+z^2}}$

$4x^2+9y^2-36 \geq 0 \Rightarrow \dfrac{x^2}{9}+\dfrac{y^2}{4} \geq 1$

Domain $=\left\{ (x,y) \;\middle|\; \dfrac{x^2}{9}+\dfrac{y^2}{4} \geq 1 \right\}$ ←



$g(x,y,z)=\dfrac{xyz}{\sqrt{x^2+y^2+z^2}}$

$x^2+y^2+z^2>0$

$(0,0,0)$

Domain $=\{ (x,y,z) \mid (x,y,z) \in R \times R \times R ,\ (x,y,z) \neq (0,0,0) \}$

4) $f_x(x,y)=x+4y$, $f_y(x,y)=3x-y$

Mixed partial derivative :- Must be equal

$f_{xy}(x,y)=4$ $f_{yx}(x,y)=3$

$f_{xy} \neq f_{yx}$.

5) $u_{xy} = u_{yx}$.

6-1) $\lim\limits_{(x,y)\to(0,0)} \dfrac{3xy}{x^2+y^2}$

$y=mx$

1) $\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2+y^2}$ $y = mx$

$$= \lim_{(x,mx)\to(0,0)} \frac{3x(mx)}{x^2+m^2x^2}$$

$m=1$, $y = x$.

$m=2$, $y=2x$

$$= \lim_{x\to 0} \frac{3m\,x^2}{(1+m^2)\,x^2}$$

$x \to 0$, $x \neq 0$.

$$= \frac{3m}{1+m^2}$$

$m=1$: $\frac{3}{2}$ $m=2$: $\frac{6}{5}$

different m value.

$\Rightarrow$ Different limit value.

Limit doesn't exist.

2) $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x+y}$

$y = -\sin x$. ← path.

$$\lim_{x\to 0} \frac{\sin(-x\sin x)}{x-\sin x}.$$ How do we find limit??

$\left(\frac{0}{0}\right)$ — inde.

$+x\sin x + \cos x$.

$$= \lim_{x\to 0} \frac{\cos(-x\sin x)\times[-x\cos x - \sin x]}{1-\cos x}.$$

$\left(\frac{0}{0}\right)$ 1 0 −2

$$= \lim_{x\to 0} \frac{-\sin(-x\sin x)[-x\cos x - \sin x]^2 + \cos(-x\sin x)\cdot[x\sin x - 2\cos x]}{\sin x}.$$

$= \infty$ Not defined.

$\left(\frac{-2}{0}\right) \neq$ Indeterminant form.

LDE.

7) $f(x,y) = \begin{cases} \dfrac{\cos y \sin x}{x}, & x \neq 0 \\ \cos y, & x = 0 \end{cases}$

f^n

$\frac{\cos x}{1} = 1$

$$\lim_{(x,y)\to(0,0)} \cos y = 1$$

$$\lim_{(x,y)\to(0,0)} \left(\frac{\cos y \sin x}{x}\right) = \lim_{(x,y)\to(0,0)} \cos y \cdot \lim_{(x,y)\to(0,0)} \left(\frac{\sin x}{x}\right)$$

$$= 1 \times 1 = 1.$$

Limit exist.

8) $D_{\vec{u}}(x,y) = \hat{n} \cdot \nabla f(x,y)$

$$\frac{2}{3} \frac{(x+y)}{\sqrt{x^2+y^2}}$$

9) $f(s,t) = te^{st}$

$\nabla f = \langle f_s, f_t \rangle$ $\nabla f(0,2) = \langle \quad \rangle$

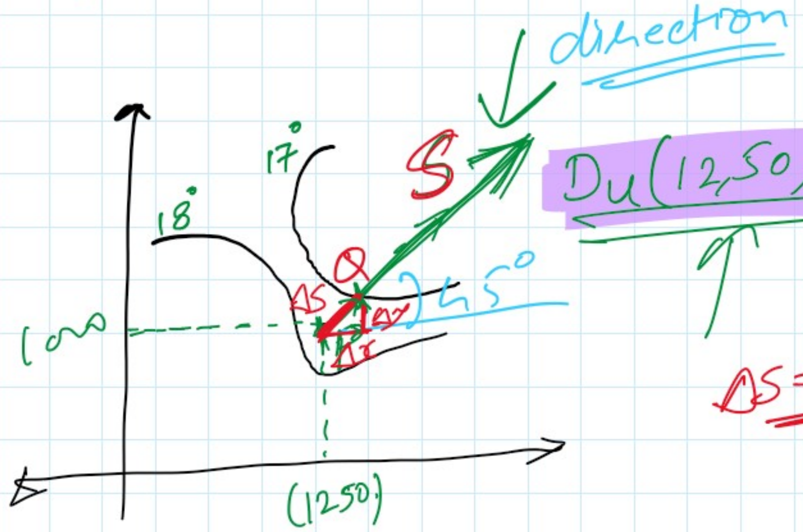
maximum rate of change. unit

$||\nabla f||$ ←

Direction must be describe in terms of unit vector

$\text{direction} = \frac{\nabla f}{||\nabla f||}$ gradient vector.

10)



$D_u(12,50, 1000)$

$\Delta S = 200$

change in temp = $-1°C$.

$\frac{\Delta T}{\Delta S} = \frac{-1}{200}$

PQ =

$= -0.005°$ per mile.

11) $f(x,y) = xy$ at $(1,2)$ Draw gradient vector.

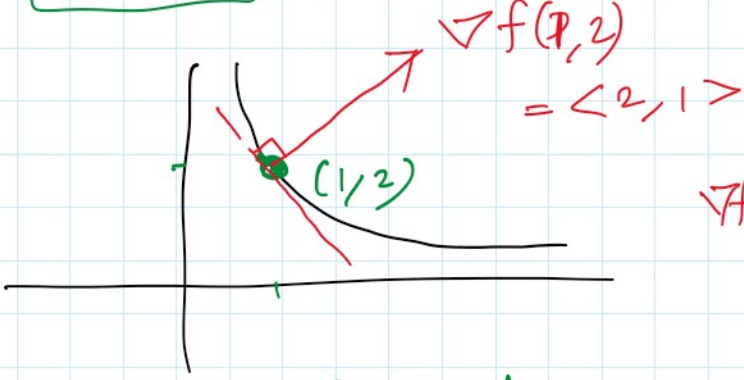
$z = xy$

$xy = 2$ ← level curve.

$y = \frac{2}{}$

$xy = 2$ ← level curve.

$y = \frac{2}{x}.$



$\nabla f(x,y) = \langle y, x \rangle$
$= \langle 2, 1 \rangle$

$(2i + 1j)$

Gradient ve

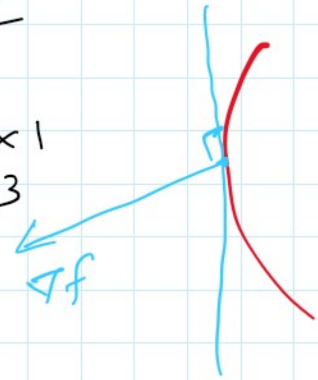
b) $\nabla f(-3,1)$ $(-3,1)$

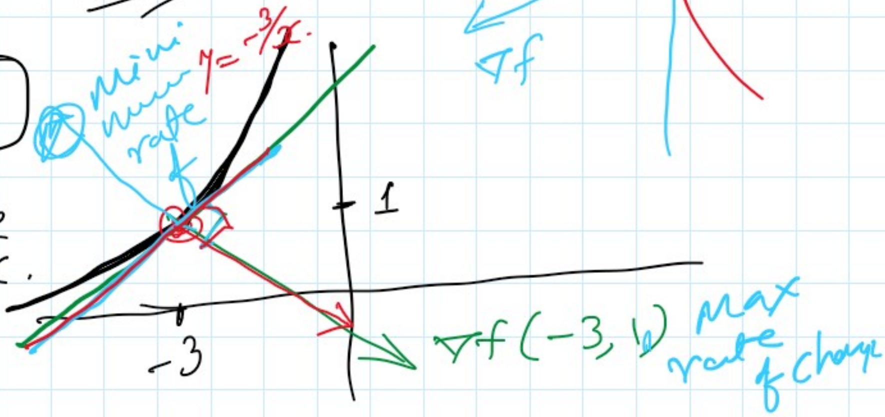
$f(x,y) = xy$

$z = xy \implies z = -3 \times 1$
$= -3$

$-3 = xy$

$y = -\frac{3}{x}.$





$f(x,y) = xy$

13) $dw = w_x dx + w_y dy + w_z dz$

$P(1,2,3)$

$w_x = 3x^2yz + y.$ 20

$w_y =$ 4

$w_z =$ 3

$$dw = 20\,dx + 4\,dy + 3\,dz$$

(4) $f(x,y) = x^2 + 4y^2 - 2x + 8y$

$x + 2y = 7$

$g(x,y) = x + 2y$

$\nabla f = \lambda \nabla g$

$D(x,y)$

f_{xx}

f_{xy}

$(5,1)$

Minimum

$f(7,0) = 35 > 27$

$f(0,3.5) = 77 > 27$

8) $f(\ ) = \sqrt{x^2+y^2}$

$f(x,y,z) = \sqrt{x^2+y^2} + 0z$