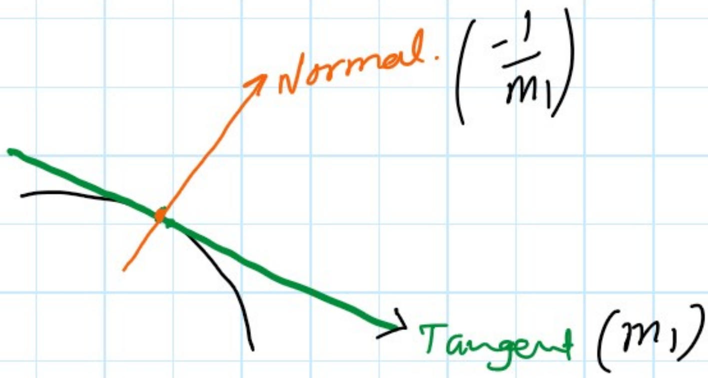
Differentiation Rule

Thursday, December 10, 2020 5:53 AM

* Tangent.
* Normal.



$$m_1 \times \frac{-1}{m_1} = -1$$

Ex The function $f(x) = 3x^2 - 2$ is given find,

a) the derivative of the f^n at $x=1$

b) The eq of tangent at $x=1$

c) The eq^n of Normal at $x=1$

Sol^n:-

$$f(x) = 3x^2 - 2$$

$$f'(x) = 6x$$

at $x=1$ $\rightarrow$ $f'(x) = 6(1) = 6$ (slope of tangent).

eq^n of tangent: point - slope:

$x=1,\ y=1,\ (1,1)$

$$y - 1 = 6(x-1)$$

$$y = 6x - 5$$

eq^n of Normal:- $(1,1),\ m_1 = \frac{-1}{6}$.

$$y - 1 = -\frac{1}{6}(x-1)$$

$$y = -\frac{x}{6} + \frac{7}{6}.$$

Ex: Find the equation(s) of any horizontal tangent to the curve $y = 2x^3 + 3x^2 - 12x + 1$.

Sol^n

$$y = 2x^3 + 3x^2 - 12x + 1$$

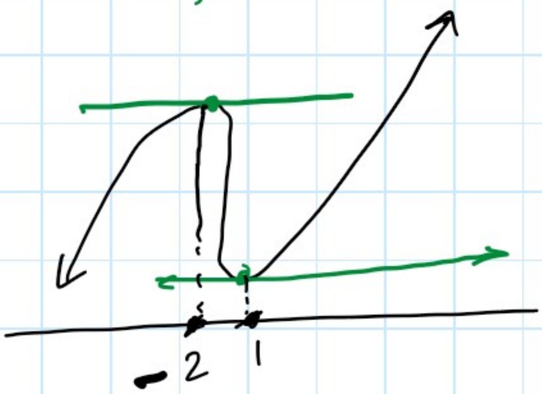
$$y' = 6x^2 + 6x - 12$$

$$6x^2 + 6x - 12 = 0$$

$$x^2 + x - 2 = 0.$$

$$x = 1 \quad , \quad x = -2$$

$$y = -6 \quad , \quad y = 21$$



$(y+6) = 0(x-1)$

$\boxed{y = -6}$

$(-2, 21)$

$y - 21 = 0(x+2)$

$\boxed{y = 21}$

Ex: Let $f(x) = \frac{-20}{x} + 1$ for $x > 0$

$g(x) = 5x + 3$ for $x \in R$

Find the values of x for which the graphs of f and g have the same gradient.

Solⁿ

$[x=2]$

$$f(x) = -\frac{20}{x} + 1$$

$f(x) = x^n$

$f'(x) = nx^{n-1}$

$$f(x) = -20x^{-1} + 1$$

$$f'(x) = -20(-1)x^{-1-1} + 0$$

$$= 20x^{-2}$$

$$f'(x) = \frac{20}{x^2} \quad \text{— I}$$

$$g(x) = 5x + 3$$

$$g'(x) = 5 + 0 \quad \text{— II}$$

$$f'(x) = g'(x)$$

$$\frac{20}{x^2} = 5$$

$$\Rightarrow x^2 = 4$$

$$x = 2, \cancel{-2}$$

(from condition).

Differentiation Rules:-

$+$
$-$
$\times$
$\div$

$$\left.\begin{aligned} y &= f(x) \pm g(x) \\ y' &= f'(x) \pm g'(x) \end{aligned}\right\} \text{I}$$

$$\left.\begin{aligned} y &= f(x) \times g(x) \\ y' &\neq f'(x) \times g'(x) \end{aligned}\right\}$$

$$\left.\begin{aligned} y &= \frac{f(x)}{g(x)} \\ y' &= \frac{f'(x)}{g'(x)} \end{aligned}\right\} \times$$

$y' \neq f'(x) \times g'(x)$ $\quad$ $y = \frac{f'(x)}{g'(x)}$

Chain Rule:

$y = (x-3)^3$ $\quad$ $\frac{dy}{dx} = ?$

$\begin{cases} f(x) = x^n \\ f'(x) = nx^{n-1} \end{cases}$

$x - 3 = u$

$y = u^3$

Differentiate w.r.t. u.

$\frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \left(\frac{du}{dx}\right)$

$u = x - 3$

$\frac{du}{dx} = 1.$

$\frac{dy}{dx} = (3u^2)(1.) = 3(x-3)^2$

$\frac{dy}{dx} = 3(x-3)^2$

Ex

$y = (x^2 - 4x)^3$ $\quad$ $\boxed{\frac{dy}{dx} = ?}$

$u = x^2 - 4x.$

* $\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.}$

$y = u^3$, $\quad u = x^2 - 4x.$

$\frac{dy}{du} = 3u^2$, $\quad \frac{du}{dx} = 2x - 4$

$y = (x^2 - 4x)^3$

$\frac{dy}{dx} = 3(x^2 - 4x)^{3-1} \cdot (2x-4)$

$= 3(x^2 - 4x)^2 \cdot (2x-4)$

$$\frac{dy}{du} = 3u^2, \quad \frac{du}{dx} = 2x-4$$

$$\frac{dy}{dx} = 3u^2 \cdot (2x-4)$$

$$= 3(x^2-4x)^2(2x-4)$$

Ex: $y = \sqrt{5x^2-2} = (5x^2-2)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(5x^2-2)^{\frac{1}{2}-1} \times (10x-0)$$

$$= \frac{\cancel{10}^{5}x}{\cancel{2}}(5x^2-2)^{-1/2}$$

$$= \frac{5x}{\sqrt{5x^2-2}}$$

Ex $y = \frac{2}{\sqrt{1-3x}} = \frac{2}{(1-3x)^{1/2}}$

$$y = 2(1-3x)^{-1/2}$$

$$n x^{n-1}$$

$$y' = \frac{dy}{dx} = \cancel{2} \times \frac{-1}{\cancel{2}}(1-3x)^{-\frac{1}{2}-1} \cdot (-3)$$

$$\begin{cases} y = K \cdot f(x) \\ y' = K \cdot f'(x) \end{cases}$$

$$= 3(1-3x)^{-3/2}$$

$$= \frac{3}{(1-3x)^{3/2}}$$

Ex $y = \frac{-3}{\sqrt{2x^2-1}}$

$$\quad\quad ^{-1/2}$$

$$= \quad \frac{\ }{\sqrt{2x^2-1}}$$

$$y = -3(2x^2-1)^{-1/2}$$

$$y' = -3 \times -\frac{1}{\cancel{2}}(2x^2-1)^{-\frac{1}{2}-1} \cdot (\cancel{4}^{2}x)$$

$$= 6x(2x^2-1)^{-3/2}$$

$$= \frac{6x}{(2x^2-1)^{3/2}}$$

product Rule:

$$y = f(x) \cdot g(x)$$

$$\boxed{y' = f(x) \cdot g'(x) + g(x) f'(x)} \quad *$$

Ex

$$f(x) = (x^3 + 3x^2 + 6) \cdot (2x-1)$$

$$f'(x) = (x^3 + 3x^2 + 6)(2) + (2x-1)(3x^2 + 6x)$$

Ex

$$y = x^2 \sqrt{x^2+1}$$

$$= x^2 \cdot \frac{d}{dx}(x^2+1)^{1/2} + \sqrt{x^2+1} \, \frac{d}{dx}(x^2)$$

$$= x^2 \cdot \frac{1}{\cancel{2}}(x^2+1)^{\frac{1}{2}-1} \cdot \cancel{2}x + \sqrt{x^2+1} \cdot (2x)$$

$$= \frac{x^3}{(x^2+1)^{1/2}} + 2x\sqrt{x^2+1}$$

#

$$y = \frac{f(x)}{g(x)}$$