Revision (Partial Derivative)

Wednesday, July 1, 2020 10:50 AM

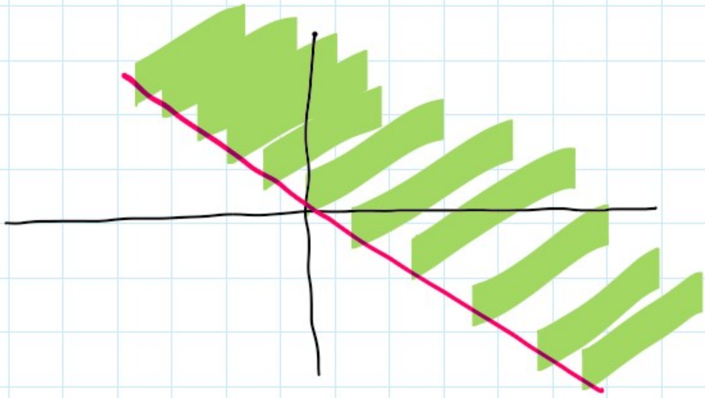
1) Determine the domain of each of the following & sketch the domain.

a) $f(x,y) = \sqrt{x+y}$

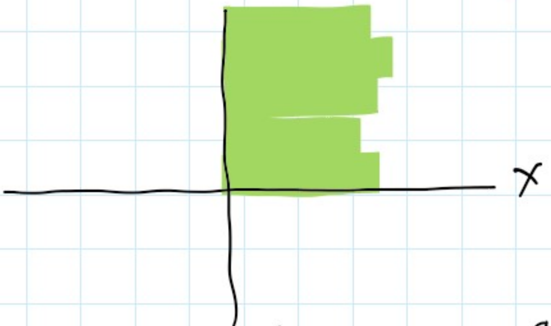
b) $f(x,y) = \sqrt{x} + \sqrt{y}$

c) $f(x,y) = \ln(9 - x^2 - 9y^2)$

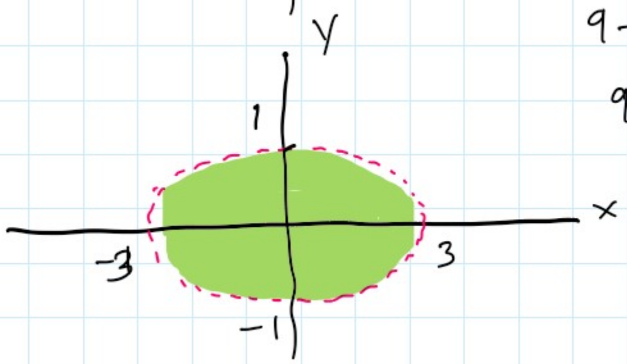
a)



b)



c)



$9 - x^2 - 9y^2 > 0$

$9 > x^2 + 9y^2$

$1 > \frac{x^2}{9} + \frac{y^2}{1}$

major minor axis

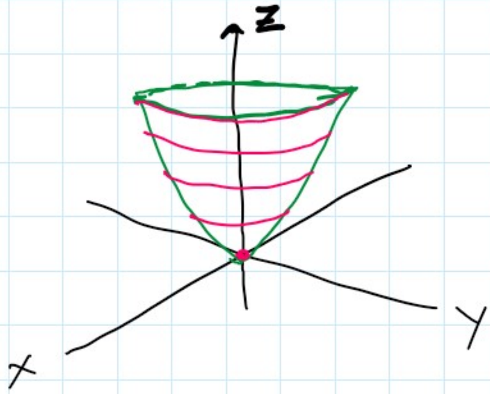
2) Determine the domain of the following fn

$$f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2-16}}$$

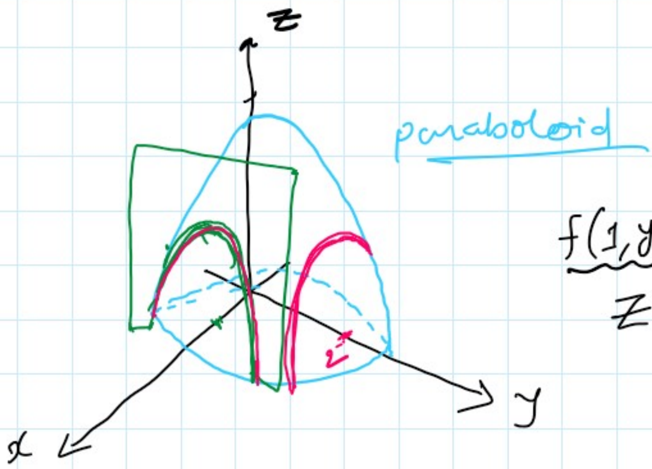
outside the sphere.

3) Identify the level curves of $f(x,y)=\sqrt{x^2+y^2}$

sketch few of them.



4) Sketch the traces of $f(x,y)=10-4x^2-y^2$ for the plane $x=1$ and $y=2$



$f(1,y)=10-4(1)^2-y^2$

$z=6-y^2$

$z=-y^2$

$z=6-y^2$

$y=2$ $\quad f(x,2)=10-4x^2-4$

$z=6-4x^2$

5) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each of the following f^n.

a) $x^3z^2-5xy^5z=x^2+y^2$

b) $x^2\sin(2y-5z)=1+y\cos(6zx)$

a) $\frac{\partial z}{\partial x}=\frac{2x-3x^2z^2+5y^5z}{2x^3z-5xy^5}$

$$\frac{\partial z}{\partial y} = \frac{2y + 25xy^4 z}{2x^3 z - 5xy^5}$$

z is implicitly defined f^n.

$z = f(x,y)$

$$x^3 z^2 - 5xy^5 z = x^2 + y^2$$

$$x^3\left(2z \cdot \frac{\partial z}{\partial y}\right) - 5x\left[y^5 \frac{\partial z}{\partial y} + z \cdot 5y^4\right] = 0 + 2y$$

$$\frac{\partial z}{\partial y}\left(2x^3 z - 5xy^5\right) - 25xzy^4 = 2y$$

$$\frac{\partial z}{\partial y} = \frac{2y + 25xzy^4}{2x^3 z - 5xy^5}$$

$$\frac{\partial z}{\partial y} = \frac{-f_y}{f_z}$$

b)

$$\frac{\partial z}{\partial x} = \frac{2x\sin(2y-5z) + 6zy\sin(6zx)}{5x^2\cos(2y-5z) - 6yx\sin(6zx)}$$

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$$\frac{\partial z}{\partial y} = \frac{\cos(6zx) - 2x^2\cos(2y-5z)}{6xy\sin(6zx) - 5x^2\cos(2y-5z)}$$

6 a) $z^2 = x^2 + y^2$, $\frac{\partial z}{\partial y} = ?\ \frac{y}{z}$ $\frac{-f_y}{f_z}$

b) $z\cos z = x^2 y^3 + z$, $\frac{\partial z}{\partial x} = ?$

$$\frac{\partial z}{\partial x} = \frac{2xy^3}{\cos z - z\sin z - 1}$$

7) Calculate $\left(\frac{\partial z}{\partial u}\right)$ and $\frac{\partial z}{\partial v}$ using the following f^n

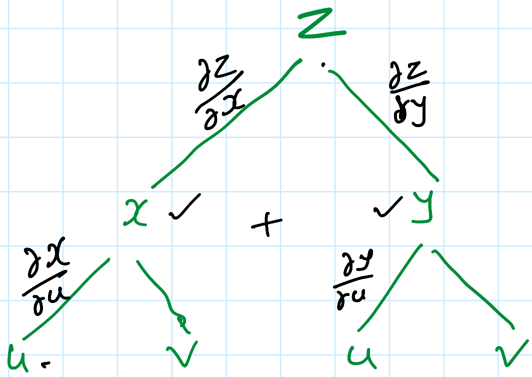
$z = f(x,y) = 3x^2 - 2xy + y^2$, $x = x(u,v) = 3u + 2v$,

$y = y(u,v) = 4u - v$

$z = f(x,y) = 3x^2 - 2xy + y^2$, $x = x(u,v) = 3u+2v$,

$y = y(u,v) = 4u - v$

$$\frac{\partial x}{\partial u} = 3$$



intermediate variable.

independent variable.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial u} = 10x + 2y$$
$$= 38u + 18v \quad *$$

$$\frac{\partial z}{\partial v} = 14x - 6y = 18u + 34v$$

8) a. Calculate $\frac{dy}{dx}$ if y is defined implicitly as a function of x via the equation

$3x^2 - 2xy + y^2 - 6y - 11 = 0$ what is the equation of tangent line to the graph of this curve at point $(2,1)$?

$$= -\frac{6x-2y}{-2x+2y-6} = \frac{y-3x}{-x+y-3} = \frac{1-3(2)}{-2+1-3} = \frac{-5}{-4} = 5/4$$

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = \cancel{\frac{3x-y+2}{x-y+3} = \frac{6-1+2}{2-1+3} = \frac{7}{4}}$$

~~$y = \frac{7}{4}x - \frac{5}{4}$~~ [Eq^n of tangent line]

$y = \frac{7}{4}x - \frac{5}{2}$ [Eq^n of tangent line]

9) Find the total differential

$$z = x^4 e^{3y} \; ; \quad dz = ?$$

$$dz = f_x \, dx + f_y \, dy$$

$$dz = 4x^3 e^{3y} \, dx + 3x^4 e^{3y} \, dy.$$