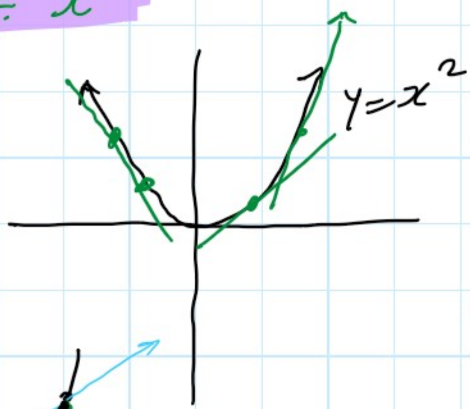
Differentiation -2

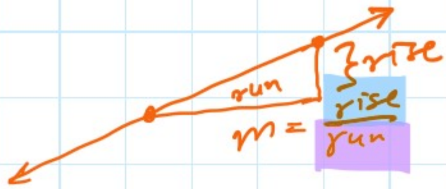
Thursday, December 3, 2020 5:52 AM

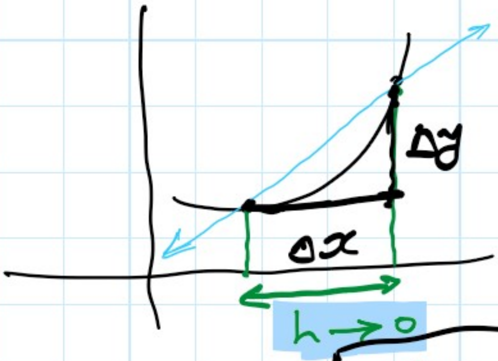
Measure change

$y = f(x) = x^2$

$f(x) = x^n$







$y = f(x)$

$$\boxed{\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - (x)}}$$

Slope of the curve.

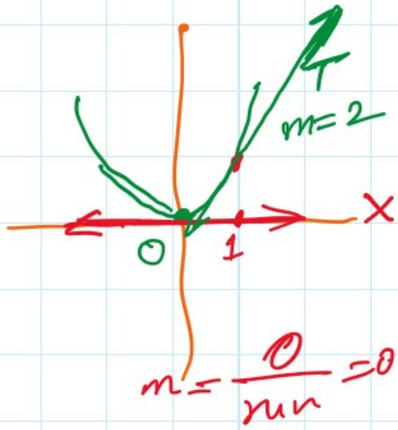
$y = x^n$

$\frac{dy}{dx} = n x^{n-1}$

$y = f(x) = x^2$

$\frac{dy}{dx} = 2x^{2-1}$

$= 2x.$



$x = 0. \Rightarrow \frac{dy}{dx} = 2(0) = 0.$

$x = 1 \Rightarrow \frac{dy}{dx} = 2(1) = 2$

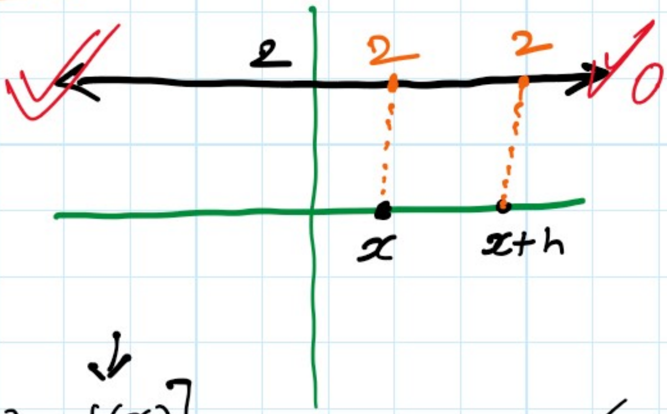
\# $\boxed{\begin{array}{l} f(x) = x^n \quad , n \in \mathbb{R}. \\ f'(x) = \frac{dy}{dx} = n x^{n-1} \end{array}}$

$f(x) = 2$

$y = 2$

$$\boxed{f'(x) = \frac{dy}{dx} = nx^{n-1}}$$

$y=2$



$\boxed{f(x) = c}$, $c \in \mathbb{R}$.

$x \longrightarrow x+h$.

$f(x+h) = c$.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{[f(x+h) - f(x)]}{(x+h) - x}.$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

$f(x) = x^n$

2# $\boxed{\begin{array}{l} f(x) = c \\ f'(x) = \dfrac{dy}{dx} = 0. \end{array}}$

3)# $\boxed{\begin{array}{l} y = k \cdot f(x) \quad , \quad k \in \mathbb{R}. \\ \dfrac{dy}{dx} = k \cdot f'(x) \end{array}}$

$y = 2(x^5)$

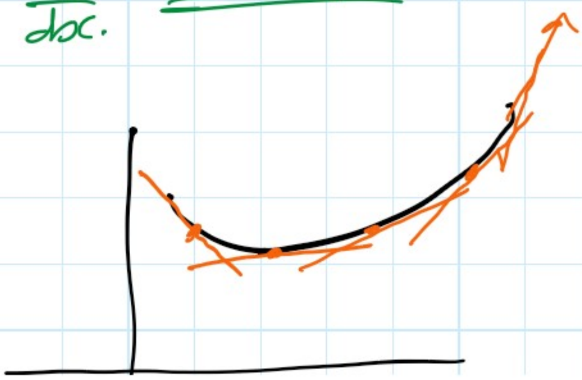
$\frac{dy}{dx} = 2 \cdot 5\, x^{5-1}$

$\frac{dy}{dx} = 10\, x^4$

Ex

$f(x) = \frac{3}{x^{12}}$

$f(x) = 3 \cdot x^{-12}$



$$f(x) = 3 \cdot x^{-12}$$

$$f'(x) = 3 \cdot (-12)\, x^{-12-1}$$

$$= \underline{\underline{-36\, x^{-13}}}$$

$$f'(x) = \frac{-36}{x^{13}}$$

$\underline{\underline{x=1}} \Rightarrow f'(1) = \frac{-36}{1^{13}} = -36.$

$\underline{\underline{x=2}} \Rightarrow f'(2) = \frac{-36}{2^{13}} = \_\_\_\_$

$\underline{\underline{Ex}}$

$$y = \underline{5} + 3x^2$$

$$\frac{dy}{dx} = \frac{d}{dx}(5) + \frac{d}{dx}(\underline{\underline{3x^2}})$$

$$= 0 + 3 \cdot \frac{d}{dx}(x^2)$$

$$= 0 + 3 \cdot 2 \cdot x^{2-1}$$

$$= 0 + 6x^1$$

$$\frac{dy}{dx} = \underline{\underline{6x}}$$

$f(x) = g(x) + h(x)$

$$\boxed{f'(x) = g'(x) + h'(x)}$$

$\underline{\underline{Ex}}$

$$f(x) = 9x^{-3}$$

$$f'(x) = 9 \times -3\, x^{-3-1}$$

$$= -27x^{-4}$$

$f(x) = \underline{x}^n$

Ex

$$f(x) = \frac{5}{x^{-3}}$$

$$\cancel{\frac{5}{-3x^{-3-1}}} \quad \times$$

$$f(x) = 5 \cdot x^{3}$$

$$f'(x) = 5 \cdot (3)\, x^{3-1}$$

$$= 15x^{2}$$

$$\boxed{\begin{aligned} f(x) &= x^{n} \\ f'(x) &= n x^{n-1} \end{aligned}}$$

Ex

$$f(x) = \frac{(2x^{4} - 3x^{3} + 1)}{x^{2}}, \quad x \neq 0.$$

$$f(x) = \frac{2x^{4}}{x^{2}} - \frac{3x^{3}}{x^{2}} + \frac{1}{x^{2}}$$

$$f(x) = 2x^{2} - 3x + x^{-2}$$

Differentiating w.r.t. x.

$$f'(x) = 4x^{1} - 3 + (-2)x^{-2-1}$$

$$= 4x - 3 - 2x^{-3}$$

Ex

$$y = (x+1)(x-2)$$

$$y = x^{2} - x - 2$$

$$\frac{dy}{dx} = 2x - 1$$

$$\left\{ \begin{aligned} f(x) &= x^{n}, \; n \in \mathbb{R} \\ f'(x) &= n x^{n-1} \end{aligned} \right.$$

Ex

$$f(x) = 6\sqrt{x}$$

find $f'(x)$

$$f(x) = 6 \cdot x^{1/2} \qquad \tfrac{1}{2} \in \mathbb{R}.$$

$$f'(x) = 3x^{-1/2}$$

$f'(x) = 3x^{-1/2}$

Ex

$f(x) = 5 \cdot \sqrt[5]{x^3} = 5 \cdot x^{3/5}$

$f'(x) = 3x^{-2/5} = 5 \cdot \frac{3}{5} \cdot x^{3/5 - 1}$ ✓

$\begin{cases} f(x) = \frac{g(x)}{h(x)} \\ f(x) = g(x) \cdot h(x) \end{cases}$

Ex find $\frac{dy}{dx}$, $y = \frac{7}{x^2} - \frac{1}{\sqrt{x}}$

$\Rightarrow y = 7 \cdot x^{-2} - x^{-1/2}$

$\Rightarrow \frac{dy}{dx} = -14x^{-3} + \frac{1}{2}x^{-3/2}$ ✓

Ex Find the gradient function ($f'(x)$) of $f(x) = x^2 - \frac{4}{\sqrt{x}}$

Hence find the gradient of the tangent to $f(x)$ at the point where $x = 4$.

Sol^n:

$f(x) = x^2 - \frac{4}{\sqrt{x}}$

Gradient function $\rightarrow f'(x) = 2x + \frac{4}{2}x^{-1/2}$

$f'(x) = 2x + 2x^{-1/2}$

$\xrightarrow{x=4}$ $f'(4) = 2(4) + 2(4)^{-1/2}$

$= 8 + 2/2$

$f'(4) = 9$

$$f'(4) = 9$$

Ex Find the co-ordinates of the point on the graph of $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 8x + 7$ where the gradient is 4.

Soln.

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 8x + 7$$

$$f'(x) = x^2 - x - 8 + 0 = 4$$

$$\Rightarrow x^2 - x - 12 = 0$$

$$\Rightarrow (x-4)(x+3) = 0.$$

$x = 4$ or $x = -3$

$$y = \frac{1}{3}(4)^3 - \frac{1}{2}(4)^2 - 8(4) + 7$$

$$y = \frac{-35}{2}$$

$$y = \frac{35}{2}$$

$\left(4, \frac{-35}{2}\right)$ $\left(-3, \frac{35}{2}\right)$

Ex $y = \frac{2x^2 - 5}{x}$ at the point $(1, -3)$

$x = 1$

$$y = \frac{2x^2}{x} - \frac{5}{x} = 2x - \frac{5}{x}$$

$$\frac{dy}{dx} = 2 + 5x^{-2}$$

$$\frac{dy}{dx} = 2 + 5x^{-2}$$

$$\left.\frac{dy}{dx}\right|_{(x=1)} = 2 + 5(1)^{-2}$$

$$= 2 + 5 = \underline{\underline{7}}$$

Eg Find the co-ordinates of the points on the graph of $f(x) = \frac{2}{3}x^3 - \frac{9}{2}x^2 - 3x + 8$ where the gradient is 2.

Soln.

$$f'(x) = 2x^2 - 9x - 3$$

$$f'(x) = 2 \Rightarrow 2x^2 - 9x - 3 = 2$$

$$2x^2 - 9x - 5 = 0$$

$$2x^2 - 10x + x - 5 = 0$$

$$2x(x-5) + 1(x-5) = 0$$

$$(2x+1)(x-5) = 0$$

$$x = -\tfrac{1}{2}, \; x = 5$$