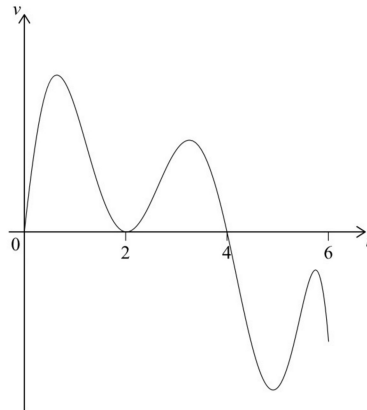# Calculus revision

5. [Maximum mark: 5]

The derivative of a function f is given by $f'(x) = 2e^{-3x}$. The graph of f passes through $\left(\frac{1}{3}, 5\right)$.

Find $f(x)$.

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7. [Maximum mark: 7]

A particle P starts from point O and moves along a straight line. The graph of its velocity, $v\,\mathrm{ms}^{-1}$ after t seconds, for $0 \le t \le 6$, is shown in the following diagram.

The graph of v has t-intercepts when $t = 0$, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_{2}^{4} v\,\mathrm{d}t = 9$.

(a)   Find the value of $s(4) - s(2)$.   [2]

(b)   Find the total distance travelled in the first 5 seconds.   [5]
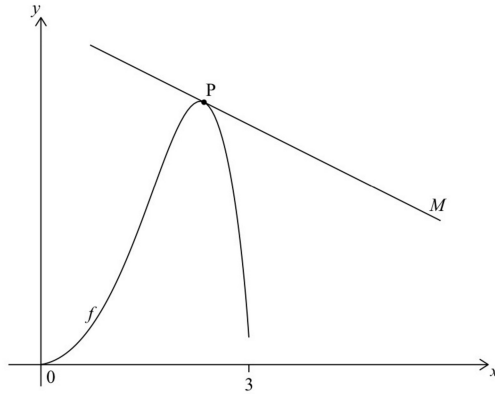
9. [Maximum mark: 15]

Let θ be an **obtuse** angle such that $\sin\theta = \dfrac{3}{5}$.

(a) Find the value of $\tan\theta$. [4]

(b) Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L. [2]

Let $f(x) = e^x \sin x - \dfrac{3x}{4}$.

(c) Find the derivative of f. [5]

The following diagram shows the graph of f for $0 \le x \le 3$. Line M is a tangent to the graph of f at point P.

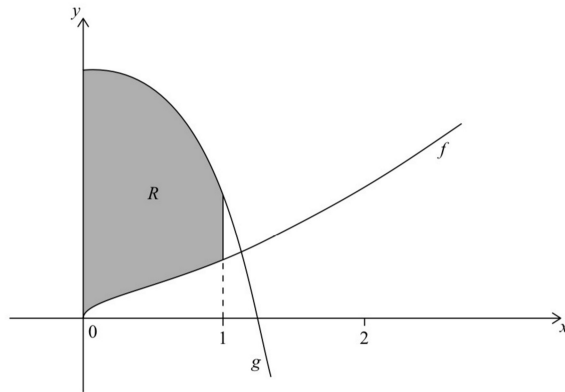(d) Given that M is parallel to L, find the x-coordinate of P. [4]

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10. [Maximum mark: 14]

Let $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

(a) Find $\dfrac{dy}{dx}$. [3]

(b) Hence find $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\ dx$. [3]

Consider the functions $f(x) = \sqrt{x^3 + x}$ and $g(x) = 6 - 3x^2\sqrt{x^3 + x}$, for $x \ge 0$.

The graphs of f and g are shown in the following diagram.

The shaded region R is enclosed by the graphs of f, g, the y-axis and $x = 1$.

(c) Write down an expression for the area of R. [2]

(d) Hence find the exact area of R. [6]

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3. [Maximum mark: 6]

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

(a) Find $f'(x)$. [4]

(b) The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. [2]

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3. [Maximum mark: 6]

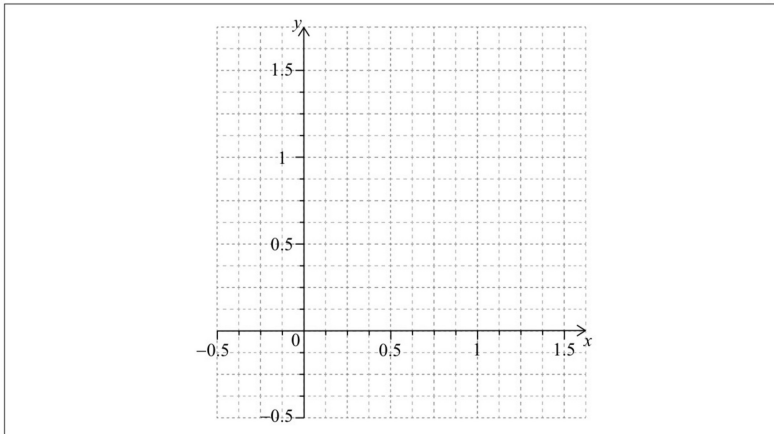Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

(a) Find $f'(x)$. [4]

(b) The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. [2]

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4. [Maximum mark: 8]

Let $f''(x) = (\cos 2x)(\sin 6x)$, for $0 \le x \le 1$.

(a) Sketch the graph of f'' on the grid below: [3]

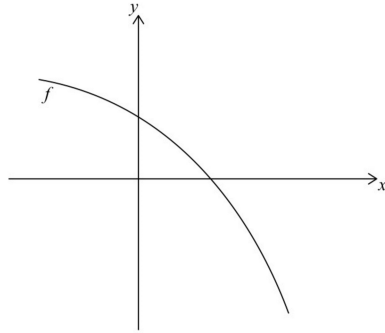(b) Find the x-coordinates of the points of inflexion of the graph of f. [3]

(c) Hence find the values of x for which the graph of f is concave-down. [2]

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2. [Maximum mark: 5]

Let $f(x) = 4 - 2e^x$. The following diagram shows part of the graph of f.

(a) Find the x-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the x-axis and the y-axis is rotated $360°$ about the x-axis. Find the volume of the solid formed. [3]

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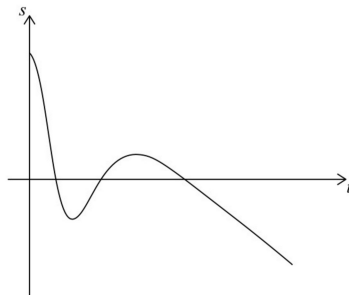..

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8. [Maximum mark: 16]

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_A = 15 - t - 6t^3 e^{-0.8t}$, $0 \leq t \leq 25$. This is shown in the following diagram.

(a) Find the initial displacement of particle A from point P. [2]

(b) Find the value of t when particle A first reaches point P. [2]

(c) Find the value of t when particle A first changes direction. [2]

(d) Find the total distance travelled by particle A in the first 3 seconds. [3]

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_B = 8 - 2t$, $0 \leq t \leq 25$.

(e) (i) Given that particles A and B start at the same point, find the displacement function s_B for particle B.

(ii) Find the other value of t when particles A and B meet. [7]