Differentiation

Thursday, November 26, 2020 5:54 AM

Limits and convergence:-

Limit of a function:-

$$\lim_{x \to 1} \left(\frac{x^2-1}{x-1} \right) = L$$

x tends to 1

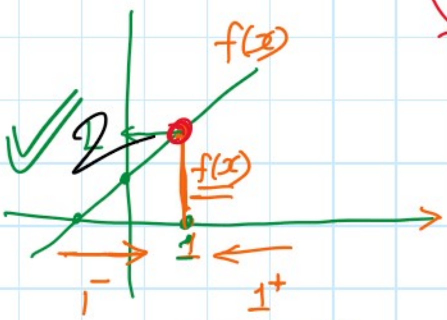
curve.

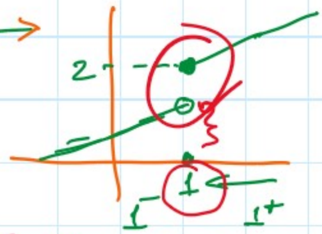
$$\lim_{x \to \infty} f(x) = a$$

$$\boxed{f(x) \neq a}$$
$$x \neq \infty$$

$$\boxed{y = a} \leftarrow \text{H. A}$$

$$f(x) = \frac{x^2-1}{x-1}$$





$x \to 1$

$$\lim_{x \to 1} \left(\frac{x^2-1^2}{x-1} \right) =$$

St. $\boxed{y = x+1.}$

$$y = \left(\frac{x^2-1}{x-1} \right)$$

$$\lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$x \to 1. \Rightarrow x \neq 1 \Rightarrow (x-1) \neq 0$$

$$\lim_{x \to 1} x+1 = \underline{2}$$

Definition of Limit:-

The limit L, of a function f exists as x approaches a real value a if and only if

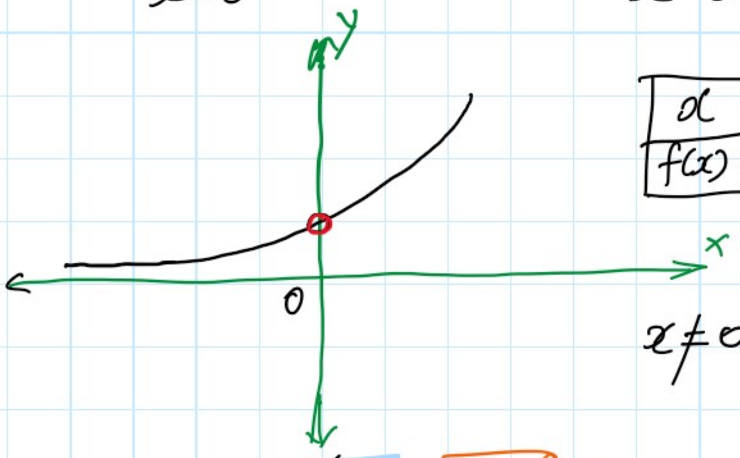
$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

we can write $\lim_{x \to a} f(x) = L$.

Ex a) Sketch $y = \dfrac{3^x - 1}{x}, \; x \neq 0$

b) $\lim_{x \to 0^-} \frac{3^x - 1}{x}$ and $\lim_{x \to 0^+} \frac{3^x - 1}{x}$.

Soln!



x	-1	-0.5	-0.01	-0.001	0
$f(x)$					

$x \neq 0$

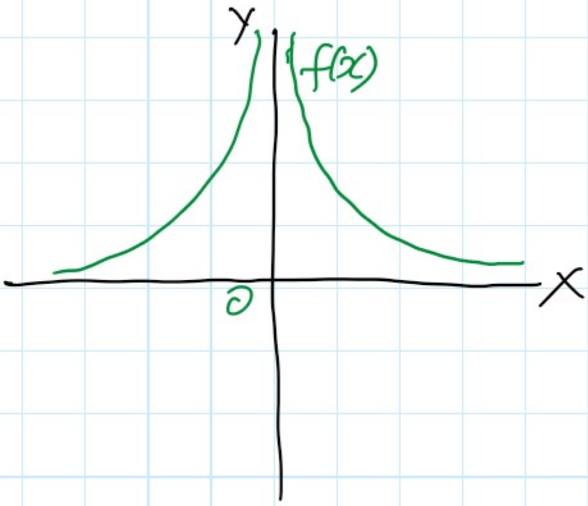
x	1	0.5	$0.25 \ldots$	0
$f(x)$				

$$\lim_{x \to 0^-} \frac{3^x - 1}{x} = \boxed{\ln 3} \approx 1.1$$

Limit at infinity.

$f(x) = \frac{1}{x^2}, \quad x \neq 0$

$$\boxed{\begin{array}{l} x \to \infty^+, \\ x \to \infty^- \end{array}} \begin{array}{l} f(x) \to 0 \\ f(x) \to 0 \end{array}$$



$y = 0$ — HA.

$x = 0$ — VA.

Ex Consider a function $f(x) = \frac{2x}{x-1}$, $x \neq 1$.

a) Sketch the f^n b) write the eq^n of Asymptotes.

$y = 2$ — HA

$x = 1$ — VA.

$y = \frac{2}{1} = 2$

finite values

HA $\left\{ \lim_{x \to \infty} f(x) = 2 \right.$

$f(x) = 2^x$.

HA $\left\{ \lim_{x \to \infty} f(x) = 2 \right\}$ ← converges

$f(x) = 2^x$

diverges

$\boxed{\begin{array}{l} x \to \infty \\ f(x) \to \infty \end{array}}$

Geometric Series.

$a, \; ar, \; ar^2, \; ar^3 \; \cdots\cdots$

Common ratio $= r$

$$\text{Sum} = S_n = \frac{a(1-r^n)}{1-r} \quad \longleftarrow \text{formula.}$$

→ Converges $\boxed{|r| < 1}$

$\boxed{-1 < r < 1}$

$5, \; 5 \times 2^1, \; 5 \times 2^2, \; 5 \times 2^3 \; \cdots\cdots$

$n \to \infty, \; S_n \to \infty$

Diverges.

$5, \; 5 \times \left(\frac{1}{2}\right)^1, \; 5 \times \left(\frac{1}{2}\right)^2, \; 5 \times \left(\frac{1}{2}\right)^3 \; \cdots\cdots$

$-1 < r = \frac{1}{2} < 1$

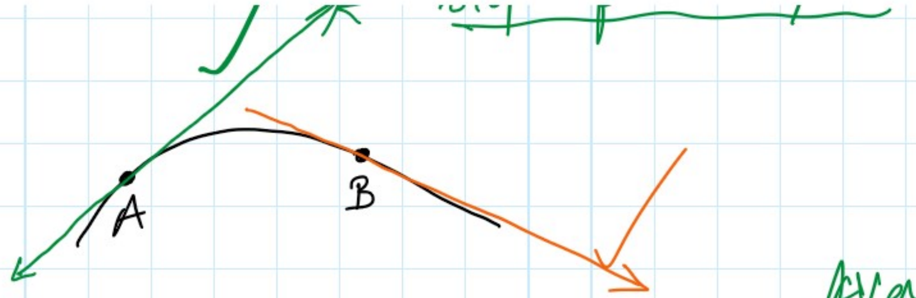
$n \to 10$ —— $S_n \to$ finite.

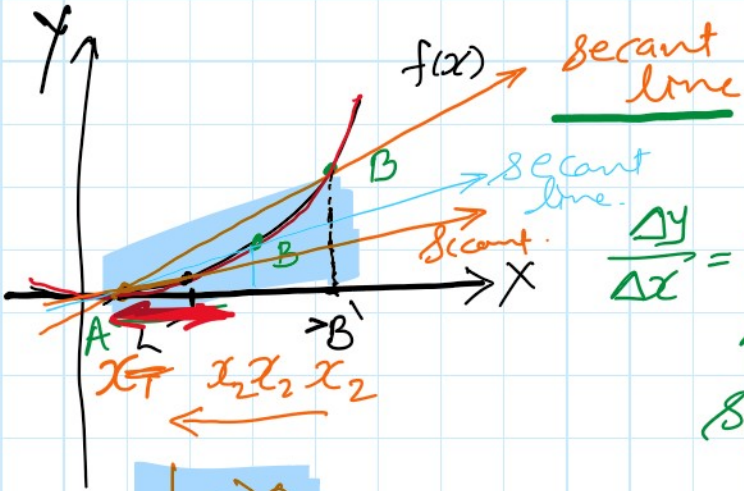
$n \to \infty \longrightarrow \boxed{S_\infty = \frac{a}{1-r}}$

$$S_\infty = \frac{5}{1 - \frac{1}{2}} = \frac{5}{1/2} = 10$$

1) Slope of the given curve:-

← Slope of the tangent.





Average rate of change.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Slope of the secant line.

$h \to 0$

$x_2 - x_1 \neq 0$

$\boxed{x_2 - x_1 = h.}$

$\boxed{h \to 0}$, $h \neq 0$.

$$x_2 - x_1 = \Delta x = h \to 0$$

$$f(x_2) - f(x_1) = ?$$

$$y_2 - y_1 = \Delta y$$

Gradient $= \frac{\Delta y}{\Delta x}$

differentiation.

$$\Delta x \to 0 \Rightarrow \boxed{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}}$$

$y = f(x) = x^n$

$f(x) = \frac{dy}{dx} = f'(x) = nx^{n-1}$

derivative of $f(x) \Rightarrow \frac{dy}{dx}$ or $f'(x) = nx^{n-1}$

first principle:

$$f'(x) = \lim_{h\to 0} \frac{\Delta y}{\Delta x} =$$

$\Delta x = h \to 0.$

$$\begin{cases} \checkmark f(x) = x^n. \\ \checkmark f(x+h) = (x+h)^n \end{cases}$$

$\Delta y = y_2 - y_1$

$x_2 - x_1 = \Delta x = h$

$\Delta x = h.$

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} \left(\frac{(x+h)^n - x^n}{\cancel{x} + h - \cancel{x}} \right)$$

$$= \lim_{h\to 0.} \frac{\left[\cancel{x^n} + nx^{n-1}h + {}^nC_2\, x^{n-2}h^2 + \cdots + h^n - \cancel{x^n}\right]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(nx^{n-1} + {}^nC_2\, x^{n-2}h + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$h \to 0$, $h \neq 0$

$$\frac{dy}{dx} \checkmark = \lim_{h\to 0} nx^{n-1} + h \text{ containing.}$$

$$\boxed{f'(x) = \boxed{nx^{n-1}}} \; ; \; f(x) = x^n$$

$f(x) = x^2$, $f'(x) = 2x^{2-1}$

$= 2x$

$f(x) = x^5$, $f'(x) = 5x^{5-1}$, Out Come

$f(x) = x^5$, $f'(x) = 5x^{5-1}$

$= 5x^4$ Out Come