Sampling distribution of the mean of repeated

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Observations of random variable

LCRV $\quad Z = aX + bY$

$z = ax + b, \quad E(Z)$

1) $E(ax+b) = a\,E(x) + b$

2) $\mathrm{var}(aX+b) = a^2\,\mathrm{var}(X)$

3) $E(aX \pm bY) = a\,E(X) \pm b\,E(Y)$

4) $\mathrm{var}(aX \pm bY) = a^2\,\mathrm{var}(X) + b^2\,\mathrm{var}(Y)$

mean/Expectation.

X – random variable. (μ, σ^2)

$x_1, x_2, x_3, x_4, \ldots\ldots x_n$ } – sample.

Mean

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\Sigma x_i}{n}$$

$$E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}E\left(\sum_{i=1}^{n} x_i\right)$$

$$= \frac{1}{n}\left[E\left(x_1 + x_2 + x_3 + \cdots + x_n\right)\right]$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + E(x_3) + \cdots + E(x_n)\right]$$

$$= \frac{1}{n}\left[\mu + \mu + \mu + \cdots + \mu\right]$$

$$= \frac{1}{n} \times n\mu = \mu$$

$$E(\bar{X}) = \mu$$

Expectation of mean of sample.

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$$Var(\bar{X}) = Var\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$
$$= \frac{1}{n^2} Var\left(\sum_{i=1}^{n} x_i\right)$$
$$= \frac{1}{n^2}\left[Var(x_1 + x_2 + x_3 + \cdots + x_n)\right]$$
$$= \frac{1}{n^2}\left[Var(x_1) + Var(x_2) + \cdots + Var(x_n)\right]$$
$$= \frac{1}{n^2}\left[\sigma^2 + \sigma^2 + \cdots + \sigma^2\right]$$
$$= \frac{1}{n^2} \cdot n\sigma^2 = \frac{\sigma^2}{n}$$

$$\boxed{Var(\bar{X}) = \frac{\sigma^2}{n}}$$

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CLT

$X \sim N(\mu, \sigma^2)$

If I take some sample out of random variable X. $(n \geq 30)$

$S_1 \sim ?(\quad , \quad)$

Q i) Find the expectation and variance of the score showing on a fair die.

ii) Find the probability distribution for the mean score when two fair dice are thrown.

iii) Find the expectation and variance of the mean score showing on two fair dice.

iv) **Show that** the expectations in **i)** and **iii)** are equal and the variance in **iii)** is half of that in **i)**.

(i)

X	1	2	3	4	5	6
$P(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$.

$E(x) = \sum x . p(x)$

$= 3.5$

$Var(x) = E(x^2) - [E(x)]^2$

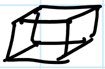
$= \frac{35}{12}$

(ii) *

	1	2	3	4	5	6	7	8	9	10	11
$\bar{x}$	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$P(\bar{X}=x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

① $\longrightarrow$ 1, 2, 3, 4, 5, 6

② $\longrightarrow$ 1, 2, 3, 4, 5, 6.



$6 \times 6 = 36$

$E(\bar{x}) = \sum x . p(x)$

$= \frac{1}{36}(1\times1 + 1.5\times2 + 2\times3 + 2.5\times4 + 3\times5 + 3.5\times6 + 4\times5 + 4.5\times4 + 5\times3 + 5.5\times2 + 6\times1)$

$= \underline{3.5}$

$Var(\bar{x}) = E(\bar{x}^2) - [E(\bar{x})]^2$

$= \frac{35}{24}$

$Var(\bar{x}) = \frac{var(x)}{\text{sample size}} = \frac{35/12}{2} = \underline{\frac{35}{24}}$

A random variable has probability distribution given by

x	1	2	3	4
p	$5k$	$2k$	k	$2k$

i) Show that $k = 0.1$ and calculate $E(X)$ and $\mathrm{Var}(X)$.

ii) If $\bar{X}$ is the mean of a randomly selected sample of 5 observations of X, write down the expectation and variance of $\bar{X}$.

i) probability distribution function (pdf)

$\therefore \quad \Sigma p = 1.$

$5k + 2k + k + 2k = 1.$

$\Rightarrow 10k = 1 \Rightarrow \underline{\underline{k = 0.1}}$

x	1	2	3	4
p	0.5	0.2	0.1	0.2

$E(X), \mathrm{var}(X) = ?$

$E(X) = 2, \quad \mathrm{Var}(X) = 1.4.$

(ii) $\boxed{n = 5}$ $\quad E(\bar{X}) = 2, \quad \mathrm{var}(\bar{X}) =$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$E(\bar{X}) = E(X) = 2, \quad \mathrm{var}(\bar{X}) = \dfrac{\mathrm{var}(X)}{5}$

$= \dfrac{1.4}{5} = \underline{\underline{0.28}}$