Extrema of Multivariable Function Part 2

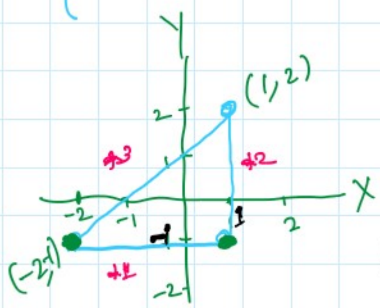
Extrema of multivariable fn:

Friday, June 26, 2020 10:58 AM

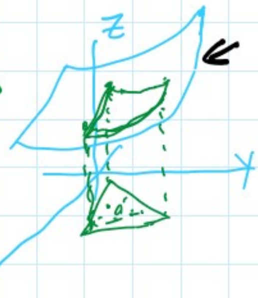
$y - y_0 = m(x - x_0)$

Ex: $f(x,y) = 3x^2 + 2xy + y^2$

R, the Δ bound by $(-2,-1), (1,-1), (1,2)$



$\left\{ m = \dfrac{2+1}{1+2} = 1 \right\}$



$(y-2) = 1(x-1)$

$y = x - 1 + 2$

$\boxed{y = x+1}$

*1

$y = -1, \quad x = x.$

$f(x,-1) = 3(x)^2 + 2x(-1) + (-1)^2$

$= 3x^2 - 2x + 1$

$f'(x,-1) = 6x - 2 = 0$

$\Rightarrow x = \frac{2}{6} = \frac{1}{3}.$

$\left.\begin{array}{l} \text{C.P. } (\frac{1}{3}, -1) \to 2/3 \\ \text{E.P. } (-2,-1) \to 17 \\ \text{E.P. } (1,-1) \to 2 \end{array}\right\}$

*2

$x = 1, \quad y = y$

$f(1,y) = 3 + 2y + y^2$

$f'(1,y) = 2 + 2y = 0$

$y = -1$

$\left.\begin{array}{l} \text{C.P. } (1,-1) = 2 \\ \text{E.P. } (1,-1) = 2 \end{array}\right\}$

E.P. $(1,2) = 11$

*3 $\quad x = x, \quad y = x+1$

$f(x, x+1) = 3x^2 + 2x(x+1) + (x+1)^2$

$= 3x^2 + 2x^2 + 2x + x^2 + 2x + 1$

$f(x, x+1) = 6x^2 + 4x + 1.$

$f'(x, x+1) = 12x + 4 = 0$

$\Rightarrow x = -\frac{1}{3}, \quad y = \frac{2}{3}$

C.P. $(-\frac{1}{3}, \frac{2}{3}) \Rightarrow 1/3$

E.P. $(-2,-1) \to 11$

E.P. $(1,2) \to 17$

How do we find CP

$\left.\begin{array}{l} f_x = 6x + 2y = 0 \\ f_y = 2x + 2y = 0 \end{array}\right\}$

$4x = 0$

$x = 0$

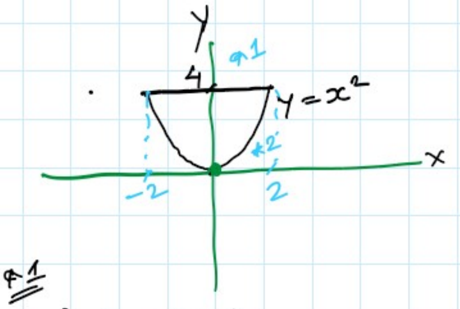
$y = 0$

C.P. $(0,0) \to 0$

$$C.P.\ (0,0) \longrightarrow 0$$

Relative Max. $\xrightarrow{(-2,-1)}$ 17
Relative Min. $\xrightarrow{(0,0)}$ 0.
} Absolute Maxima & Minima.

Ex: $f(x,y) = xy - x^2$ on R bound by $y = x^2$ & $y = 4$



$f_x = y - 2x = 0$
$y = 0$
$f_y = x = 0$
$C.P.\ (0,0) \rightarrow 0$

***1**

$y = 4,\ x = x.$

$f(x,4) = 4x - x^2$
$f'(x,4) = 4 - 2x = 0$
$x = 2,\ y = 4$

$C.P.\ (2,4) \longrightarrow 4$
$E.P.\ (2,4) \rightarrow 4$
$E.P.\ (-2,4) \rightarrow -12$

***2:** $y = x^2,\ x = x.$

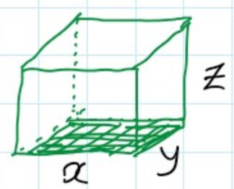
$f(x,x^2) = x \cdot x^2 - x^2$
$= x^3 - x^2$
$f'(x,x^2) = 3x^2 - 2x = 0$
$x = 0$ or $x = 2/3$
$\downarrow \qquad \downarrow$
$y = 0 \qquad y = \frac{4}{9}$

$C.P.(0,0) \rightarrow 0 \qquad C.P.\left(\frac{2}{3}, \frac{4}{9}\right) \rightarrow \frac{-4}{27}$

Abs. Max. is 4 at $(2,4)$
Abs. Min is -12 at $(-2,4)$

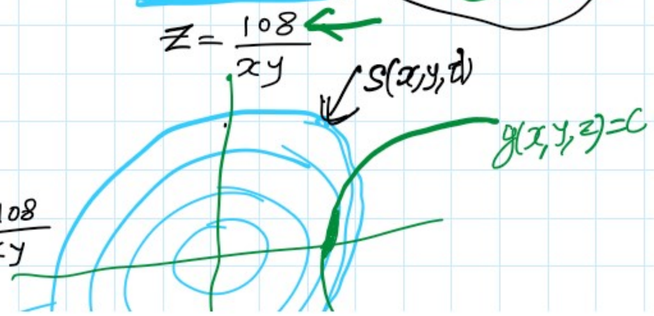
Ex: Minimize the material needed to create an open box with a volume of $108\ in^3$

Soln:



Constrained:
$\boxed{xyz = 108}$ $\leftarrow$ $g(x,y,z) = c$

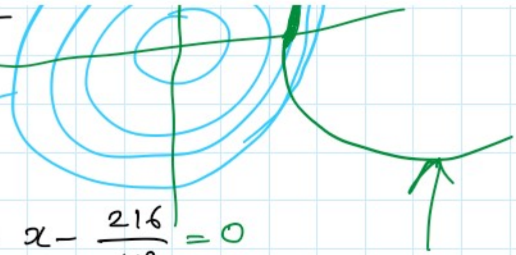
$z = \frac{108}{xy}$



$S = xy + 2yz + 2xz$

$S = xy + 2y \times \frac{108}{xy} + 2x \cdot \frac{108}{xy}$

$$S = xy + 2y \times \frac{108}{xy} + 2x \cdot \frac{108}{xy}$$



$$S = xy + \frac{216}{x} + \frac{216}{y}$$

$$S_x = y - \frac{216}{x^2} = 0 \quad ; \quad S_y = x - \frac{216}{y^2} = 0$$

$$y = \frac{216}{x^2} \implies x - \frac{216}{\left(\frac{216}{x^2}\right)^2} = 0$$

$$x - \frac{x^4}{216} = 0$$

$$x\left(1 - \frac{x^3}{216}\right) = 0$$

$x = 0$, or $1 - \frac{x^3}{216} = 0$

X invalid.

$$x = 6$$

$$y = \frac{216}{36} = 6, \quad (6,6)$$

$$z = \frac{108}{xy} = \frac{108}{6 \times 6} = \frac{108}{36} = 3$$

$$(6,6,3)$$

$$D(x,y) = S_{xx} S_{yy} - (S_{xy})^2$$

$$S_{xx} = \frac{216 \times 2}{x^3} \xrightarrow{x=6} +ve$$

$$(6,6,3) \leftarrow$$

Lagrange's Multiplier: (λ)

concept:

$f(x,y) = z$ (surface)

$f(x,y) = c_1$ or c_2 or c_3

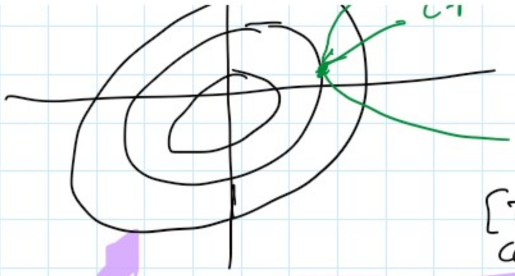
constrained

$g(x,y) = c$

$$\boxed{z = g(x,y)}$$

$z = c$

$g(x,y) = c$



$z = c$

$g(x,y) = c$

level curve.

[The intersection gives constrained Max or minim.

level curve of $f(x,y) = c_1 / c_2 / c_3$ etc.