

Tangent Plane and Normal Line

Monday, June 22, 2020 10:56 AM

23.085

Chain Rule: $\begin{matrix} 0.01 & 21.37 \\ \nearrow & \searrow \\ \frac{dT}{dt} = \frac{\partial T}{\partial c} \left(\frac{dc}{dt} \right) + \frac{\partial T}{\partial s} \left(\frac{ds}{dt} \right) \end{matrix}$

$$T(c,s) = 57s^2 \sqrt{0.95+c}$$

$$\frac{\partial T}{\partial c} = \frac{57s^2}{2\sqrt{0.95+c}}, \quad \frac{\partial T}{\partial s} = \sqrt{0.95+c} \cdot (104)s$$

$$s = 1, c = 0.5, T(c,s) \approx 57$$

2)

2.74

$$57 \pm 2.74$$

$$T(x,y) = \frac{120x^2}{x^2+y^2}$$

5)

$$\nabla f(2,2) = \langle f_x, f_y \rangle$$

$$\nabla f(x,y) = \left\langle \frac{240xy^2}{(x^2+y^2)^2}, \frac{-240x^2y}{(x^2+y^2)^2} \right\rangle$$

At (2,2) \Rightarrow

$$\nabla f(2,2) = \langle 30, -30 \rangle$$

Max rate of change = $30\sqrt{2}$

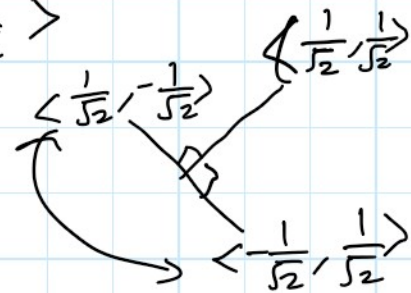
Direction is mentioned in terms of unit vector always.

$$\begin{aligned} \hat{u} &= \frac{\langle 30, -30 \rangle}{\sqrt{900+900}} = \frac{\langle 30, -30 \rangle}{30\sqrt{2}} \\ &= \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle \\ &= \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle \end{aligned}$$

6)

$$\begin{aligned} \nabla f(1, -1) &= \left\langle \frac{240}{4}, -\frac{240(-1)}{4} \right\rangle \\ &= \langle 60, 60 \rangle \end{aligned}$$

$$\hat{v} = \frac{\langle 60, 60 \rangle}{\sqrt{2 \times 60^2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$



Required direction:-

a) $\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

b) $\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

