Tangent plane normal line

Wednesday, June 17, 2020 11:03 AM

1) $\frac{-2}{25}\,\hat{i} + \left(\frac{-4}{25}\right)\hat{j} \quad : \begin{pmatrix} -2/25 \\ -4/25 \end{pmatrix}$

2) $\frac{7}{5\sqrt{3}} = \frac{7\sqrt{3}}{15}$ ✓

$f(x,y) = (x^2+y^2+1)^{1/2}$ at $P(1,1)$

3) $-\frac{7\sqrt{13}}{13}$

$f(x,y) = xe^{-y}$ $P(2,0)$

Directn. $P(2,0)$ $Q(-1,2)$

$\bar{v} = \langle -3, 2\rangle$

$\hat{u} = \frac{\bar{v}}{\|v\|} = \frac{\langle -3, 2\rangle}{\sqrt{13}}$

4) $1/6$

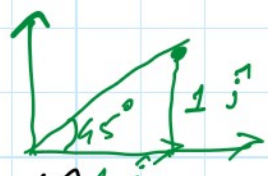
Q find the direction in which $f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$

a) increases most rapidly at the point (1,1) and

b) decreases most rapidly at (1,1)

c) what are the directions of zero change in f at (1,1)?

Solⁿ: a) $\nabla f_{(x,y)} = f_x\,\hat{i} + f_y\,\hat{j}$



$\nabla f(x,y) = x\,\hat{i} + y\,\hat{j}$ ✓

$(1,1) \Rightarrow \nabla f_{(1,1)} = \hat{i} + \hat{j} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\hat{u} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ Direction must be described in unit vector

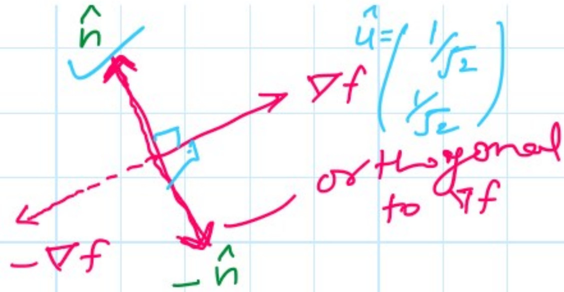
b) $-\nabla f_{(1,1)} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\hat{n}$

$\hat{u} = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

$\rightarrow \nabla f$ $\hat{u} = \left(\frac{1}{\sqrt{2}} \right.$

$$\hat{u} = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$$

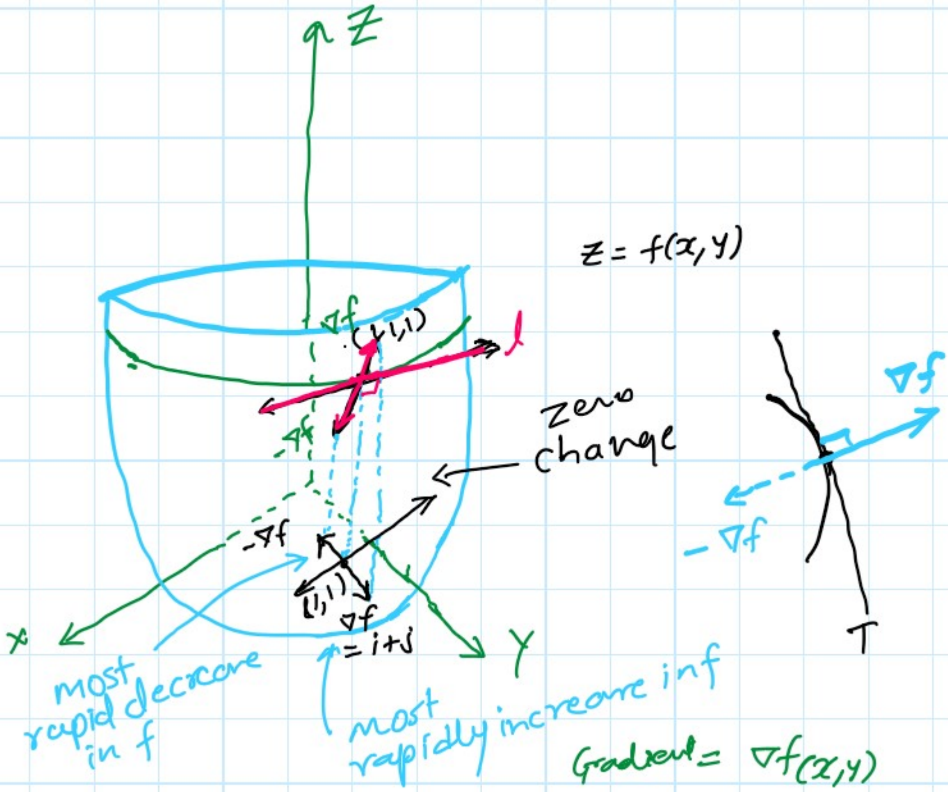


c)

$$\hat{n} = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

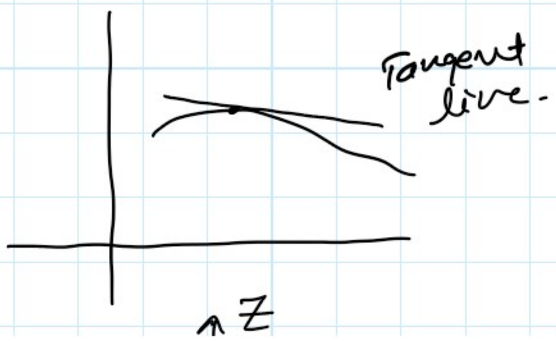
$$-\hat{n} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

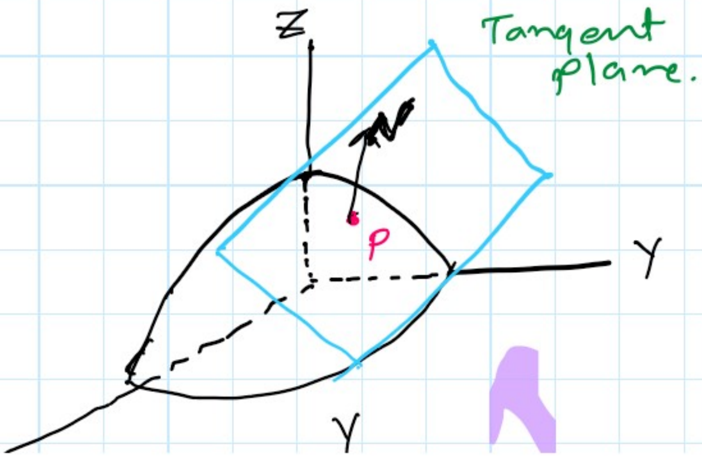
$$\begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = -\frac{1}{2} + \frac{1}{2}$$
$$= 0.$$

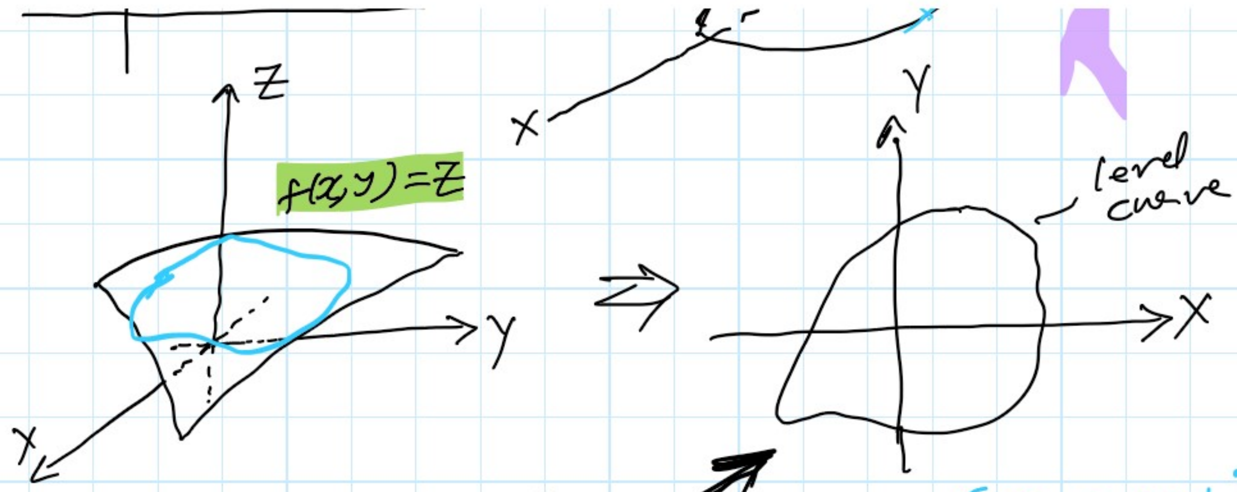


Tangent planes & Normal lines:-

tangent plane to a surface at a point







$\gamma(t) = \boxed{x(t), y(t)}$

t independent variable
x, y are intermediate var.

$$\left\{\begin{array}{l} x = a\cos t \\ y = a\sin t \\ \text{circle} \end{array}\right\}$$

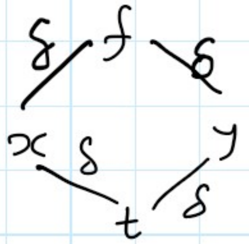
$f(x(t), y(t)) = \underline{\underline{C}}$

Take the derivative.

$f'(x(t), y(t)) = 0$

$$\left\{\begin{array}{l} f(x,y) = Z \\ \text{what is level curve} \\ \text{at } Z, C = 4 \end{array}\right.$$

chain Rule.



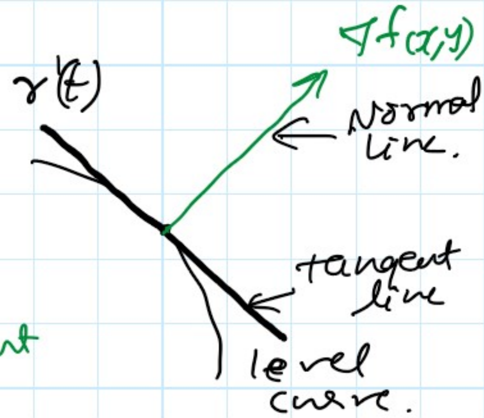
$$\frac{\partial f}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} = 0$$

$$\Rightarrow \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}\right)\cdot\left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right) = 0.$$

$$\Rightarrow \left(f_x\hat{i} + f_y\hat{j}\right)\cdot\left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right) = 0.$$

$$\Rightarrow \nabla f(x,y)\cdot\gamma'(t) = 0.$$

what is this:-



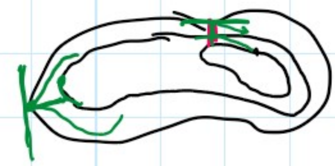
1) $\bar{\gamma}'(t)$ gives a slope of tangent

1) $\bar{r}'(t)$ gives a slope of tangent vector of level curve.) level curve.

2) when a dot product $=0$, the two vectors are orthogonal.

3) So, ∇f gives us the normal to a level curve at a point.

The fastest way up a hill is along the path $\perp$er to a level curve. ∇f



$\nabla f(x,y)$ is the normal to a level curve $f(x,y)=C$

$\nabla f(x,y,z)$ is the normal to a level surface $f(x,y,z)=C$

Ex: Find the tangent line & normal to $x^2-y^2=16$ at $P(5,3)$

Sol^n:

$f(x,y)=C.$

$\boxed{f(x,y)=x^2-y^2}$

we pretend that $x^2-y^2=16$ is a level curve to some surface in R-3

So, $f(x,y)=x^2-y^2$

$x^2-y^2=16$ is a level curve to $f(x,y)$.

means $\nabla f(x,y)$ gives normal to $x^2-y^2=16$.

$\nabla f(x,y) = 2x\,\hat{i} - 2y\,\hat{j}$ ← Normal vector to any level curve $x^2-y^2=C$

$\nabla f(5,3) = 10\,\hat{i} - 6\,\hat{j}$ ← the $P(5,3)$ give a specific normal to a specific level

$\nabla f(5,3) = \underline{10\,i - 6\,j}$ $\leftarrow$ the p(5,3) gi[illegible] normal to a specific level curve $(x^2 - y^2 = 16)$

$m_N = \frac{-6}{10} = \frac{-3}{5}$

Normal line: ??

Eqn:

$$y - 3 = \frac{-3}{5}(x-5)$$

$$\boxed{y = -\frac{3}{5}x + 6.}$$

Now,

$M_T = \frac{5}{3} \Rightarrow y - 3 = \frac{5}{3}(x-5)$ — Eqn of tangent

$$\boxed{y = \frac{5}{3}x - \frac{16}{3}}$$