Linear combination of poisson and normal variables

Wednesday, January 13, 2021 4:52 PM

$Y = ax + b$ ← Normal distribution. X - normal distribution

$S = ax + bY$ ← poisson distribution. X -, Y - poisson distribution.

The sum of two independent poisson random variable is still a poisson:..

If X & Y are two independent poisson random variables & $X \sim Po(\underbrace{\lambda}_{\text{mean, variance}})$, $Y \sim Po(\mu)$

$X+Y \sim Po(\lambda + \mu)$

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

Ex

$X \sim Po(2.3)$

$Y \sim Po(3.8)$

$X+Y \sim Po(6.1)$

$$P(X+Y=3) = \frac{e^{-6.1} \cdot (6.1)^3}{3!}$$

For the sum to be $\underline{3}$

$P(X+Y=3) = ?$

$X=0, Y=3$

$X=1, Y=2$

$X=2, Y=1$

$X=3, Y=0$

$$P(X+Y=3) = P(X=0, Y=3) + P(X=1, Y=2) + P(X=2, Y=1) + P(X=3, Y=0)$$

$$= e^{-2.3} \cdot \frac{e^{-3.8} \cdot 3.8^3}{3!} + \frac{e^{-2.3} \cdot 2.3}{1!} \cdot \frac{e^{-3.8} \cdot (3.8)^2}{2!}$$

$$+ \frac{e^{-2.3} \cdot (2.3)^2}{2!} \cdot \frac{e^{-3.8} \cdot (3.8)}{1!} + \frac{e^{-2.3} \cdot (2.3)^3}{3!} \times e^{-3.8}$$

$$= \frac{e^{-6.1} (6.1)^3}{3!}$$

Ex

Two radioactive substances emit, on average, 3 electrons per second and 2 electrons per second respectively.

Calculate the probability that a total of exactly 4 electrons are emitted in a given second.

$X \sim Po(3)$, $Y \sim Po(2)$

$S = X + Y$

$S \sim Po(5)$

$$P(X+Y=4) = \frac{e^{-5} \cdot (5)^4}{4!}$$

Ex

The number of goals a team scores in a league match may be modelled by a Poisson distribution with mean 2.2. The number of goals the same team concedes in a league match may be modelled by a Poisson distribution with mean 1.5.

a) Assuming these are independent of one another, find the probability that:
   i) the match finishes as a 1–1 draw
   ii) there are two goals in the match.

b) Comment on the assumption that the number of goals scored and conceded by a team are independent of one another.

Sol^n:

a)(i) $P(\text{Score } 1\text{-}1) = \dfrac{e^{-2.2} \cdot 2.2}{1!} \times \dfrac{e^{-1.5} \cdot 1.5}{1!} = 0.0816$

(ii) $P(\text{two goals}) = \dfrac{e^{-3.7} \cdot 3.7^2}{2!} = 0.169$

b) → 3-1, 2-1

2-2, 2-1

Linear functions and combinations of normal random variables:

If $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$ are independent.

then.

(1) $S = ax+b \sim N(a\mu_x + b,\ a^2 \sigma_x^2)$

(2) $S = ax+by \sim N(a\mu_x + b\mu_y,\ a^2 \cdot \sigma_x^2 + b^2 \sigma_y^2)$

Ex

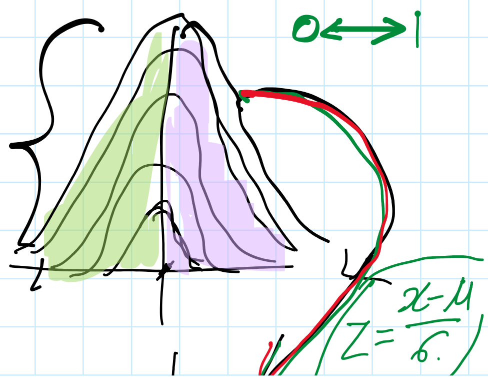
A container is known to have mass 40 grams. The amount of liquid a machine dispenses into the container follows a normal distribution with mean 200 ml and standard deviation 10 ml.
The liquid has density 0.85 kg per litre.
Find the probability that the filled container weighs less than 200 grams.

{Standard normal distribution.}

container = 40 gms

$X \sim N(200, 10^2)$

$Y = \underset{\text{gms.}}{40} + \underset{\text{gms.}}{0.85X}$



$Y = \underset{gms.}{40} + \underset{gms.}{0.85X}$

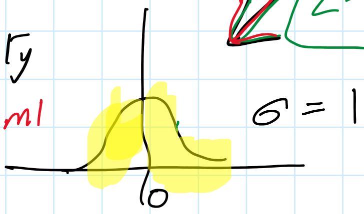
$Z = \frac{x-\mu}{\sigma}$

weight of liquid = volume × density

$= (X)\, 0.85 \frac{kg}{lt} \times ml$

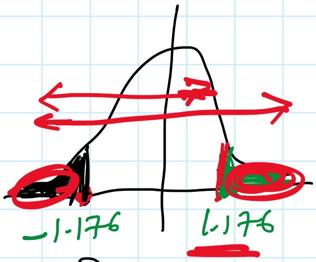
$= 0.85 \times 10^3 \times g \frac{ml}{lt}$

$= 0.85g$



$Y \sim N(0.85 \times 200 + 40,\ 0.85^2 \times 10^2)$

$Y \sim N(210,\ 8.5^2)$



$P(Y < 200) = P\left\{ Z < \frac{200-210}{8.5} = -1.176 \right\}$

$(1 - \phi(1.176))$

$= \underline{\underline{0.120}}$

$= \underline{\underline{12\%}}$

① The mean of n independent normal random variable is still normal distribution.

$Y = \overline{X}_n = \frac{\Sigma x_i}{n}$

$Y = \overline{X}_n \sim N\left(\mu_x,\ \frac{\sigma_x^2}{n}\right)$

$X \sim N(\mu_x, \sigma_x^2),\ Y \sim N(\mu_y, \sigma_y^2)$

$X - Y \sim N(\underset{\text{mean}}{\mu_x - \mu_y},\ \underset{\text{vari}}{\sigma_x^2 + \sigma_y^2})$

Ex

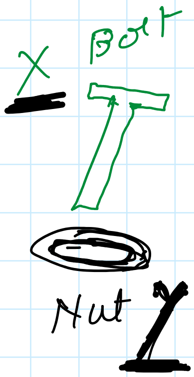
Ex

A batch of bolts has diameters which are normally distributed with mean 17.5 mm and standard deviation 0.4 mm. The diameters of holes of the batch of nuts delivered with the bolts are normally distributed with mean 18 mm and standard deviation 0.3 mm.

i) Find the probability that a randomly chosen bolt will fit inside a randomly chosen nut.

If the diameter of the bolt is more than 1.1 mm smaller than the diameter of the hole then the bolt is not securely held.

ii) Find the probability that a randomly chosen bolt which fits inside a randomly chosen nut will be held securely.



Sol^n:

$X \sim N(17.5, 0.4^2) \leftarrow$ Bolts

$Y \sim N(18, 0.3^2) \leftarrow$ Nuts.

$Y - X > 0$

diam $Y >$ diam X

$P(Y - X > 0) = ?$

$0.4^2 + 0.3^2$

$Y - X \sim N(0.5, \ 0.25)$

$\sqrt{0.25} = 0.5$

$Y - X \sim N(0.5, 0.5^2)$

$P(Y-X>0) = P\left\{ Z > \frac{0 - 0.5}{0.5} = -1 \right\}$

$= 0.8413$ ✓

(ii)

$P(0 < Y - X < 1.1) = P\left(-1 < Z < \frac{1.1 - 0.5}{0.5} = 1.2\right)$

$= 0.7262$

$P(0 < Y - X < 1.1 \,/\, Y - X > 0) = \frac{P\{(0 < Y - X < 1.1) \cap (Y - X > 0)\}}{P(Y - X > 0)}$

$= \frac{0.7262}{0.8413}$