Binomial theorem

Thursday, October 15, 2020 6:11 AM

No. of terms = 3

Expansion.

$(a+b)^2 = 1a^2 + 2ab + b^2$ $a\,(2,1,0)$ $b\,(0,1,2)$ $n=2$

2   1+1   2

Binomial Expression.

$5^3 = 5\times5\times5 = 5^2\times5$

$n=3$

$(a+b)^3 = (a+b)^2(a+b)$ $a\,(3,2,1,0)$ $b\,(0,1,2,3)$

$= (a^2+2ab+b^2)(a+b)$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + b^3$ $n=3$

3   2+1   1+2   3

$(a+b)^4 = 1a^4 \quad 4a^3b \quad 6ab^3 \quad 4ab^3 \quad 1.b^4$

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^1 = a+b$

$(a+b)^0 = 1.$

2D triangle } Pascal's triangle

n=0	1
n=1	1 1.
n=2	1 2 1
n=3	1 3 3 1.
n=4	1 4 6 4 1.
n=5	1 5 10 10 5 1.

$(a+b)^{10} =$

$(a+b)^{25} =$

$(a+b)^{10} = a^{10} + a^9b + a^8b^2 + a^7b^3 + a^6b^4 + a^5b^5 + a^4b^6 + a^3b^7 + a^2b^8 + ab^9 + b^{10}$

combination:-

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

factorial

$2! = 2\times1$

$3! = 3\times2\times1.$

$4! = 4\times3\times2\times1 = 4\times3!$

$r =$ 1 less than the no. term.

$r = 1-1 = 0.$

$r = 2-1 = 1.$

$${}^{10}C_5 = \frac{10!}{5!\,(10-5)!} = \frac{10!}{5!\times5!} = \frac{10\times9\times8\times7\times6\times\cancel{5!}}{5!\times\cancel{5!}}$$

$$= \frac{\cancel{10}\times9\times\cancel{8}\times7\times\cancel{6}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$

$\boxed{9\times4\times7 =}$

$r=5$

No. of term $= 5+1 = 6^{th}.$

${}^{10}C_8$

coefficient of 9th term.

$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$1! = (0! = 1)$

$\boxed{n=5}$ terms

5   5!   5!

$1! = (0! = 1)$

$\boxed{n=5}$

	terms	
$r=0$	1st	${}^5C_0 = \frac{5!}{0!\,(5-0)!} = \frac{\cancel{5!}}{0!\,\cancel{5!}} = 1.$
$r=1$	2	${}^5C_1 = \frac{5!}{1!\,(5-1)!} = \frac{5!}{1!\,4!} = \frac{5\times 4!}{4!} = 5$
$r=2$	3rd	${}^5C_2 = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\,3!} = \frac{5\times \overset{2}{\cancel{4}}\times \cancel{3!}}{\cancel{2}\times \cancel{3!}} = \underline{\underline{10}}$
$r=3$	4th	${}^5C_3 = \frac{5!}{3!\,2!} = \underline{\underline{10}}$
$r=4$	5th	${}^5C_4 = \frac{5!}{4!\times(5-4)!} = \frac{5!}{4!\times 1!} = 5$
$r=5$	6th	${}^5C_5 = \frac{5!}{5!\,(5-5)!} = \frac{5!}{5!\,0!} = 1.$

$${}^5C_0 = {}^5C_5 \quad , \quad {}^5C_2 = {}^5C_3$$

$${}^5C_1 = {}^5C_4$$

${}^5C_1 = {}^5C_{5-1}$

$${}^5C_r = {}^5C_{\underline{5-r}}$$

${}^5C_2 = {}^5C_{5-2}$

$${}^nC_r = {}^nC_{n-r}$$

$\underline{\underline{\text{Ex}}}$ Expand $(2x+1)^4$ in descending powers of x and simplify your answer.

$\underline{\underline{n=4}}$ no. of terms $= 5$

$$\underline{(2x+1)^4} = {}^4C_0(2x)^4(1)^0 + {}^4C_1(2x)^3(1)^1 + {}^4C_2(2x)^2(1)^2 + {}^4C_3(2x)^1(1)^3 + {}^4C_4(2x)^0(1)^4$$

first term $= 2x$

Second term $= 1$

$${}^4C_0 = \frac{4!}{0!\times 4!} = 1.$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

Coefficient: $\underline{1 \quad 4 \quad 6 \quad 4 \quad 1.}$

coefficients   1 4 6 4 1

$^nC_r = \frac{n!}{r!(n-r)!}$

$(2x+1)^4 = (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x)^1 + 1$

$= 16x^4 + 32x^3 + 24x^2 + 8x + 1.$

$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$ (ascending)

Ex Expand $(3x-2)^5$ in descending powers of x and simplify your answer.

First term $= 3x$

second term $= -2$

$(3x-2)^5 = 243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$

V. good

Ex Consider the expansion of $(x+2)^5$

a) write down the number of terms in this expansion. 6

b) Find the term containing x^3.

3rd

$(x+2)^5$

x^3

$n=5$

$x^5 \quad x^4 \quad x^3$

$^5C_0 \quad ^5C_1 \quad ^5C_2$

$n-r=3$

$5-r=3$

$5-3=r$

$\boxed{r=2}$

3rd term.

Sigma

$$\boxed{(a+b)^n = \sum_{r=0}^{n} {^nC_r}\, a^{n-r} b^r}$$

1st term, $r=0$

$^nC_0\, a^{n-0} b^0$

$= a^n b^0 = a^n$

3^{rd} term.

$= a^n b^0 = a^n$

$$^5C_2 = \frac{5!}{2!\,3!} = 10.$$

5

$$10 \cdot (x)^3 (2)^2$$

$$\boxed{= 40x^3}$$