

Binomial theorem

Thursday, October 15, 2020 6:11 AM

No. of terms = 3

Expansion.

$$(a+b)^2 = a^2 + 2ab + b^2$$

Binomial Expression.

$n=2$

$a(2,1,0)$
 $b(0,1,2)$

$5^3 = 5 \times 5 \times 5 = 5^2 \times 5$

$$(a+b)^3 = (a+b)^2 (a+b)$$

$$= (a^2 + 2ab + b^2)(a+b)$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$n=3$

$a(3,2,1,0)$
 $b(0,1,2,3)$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

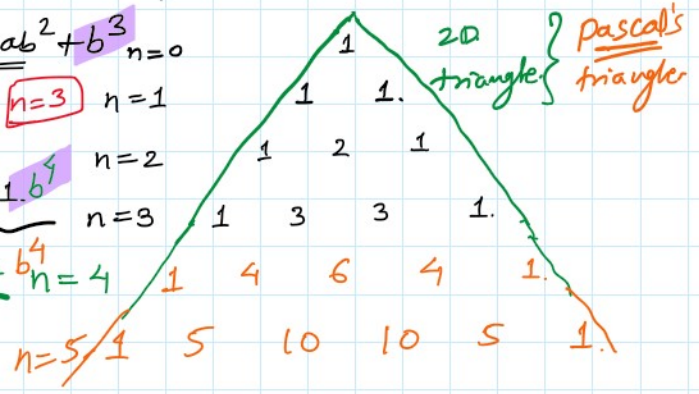
$n=4$

$(a+b)^5 = \dots$

$(a+b)^{25} = \dots$

$$(a+b)^1 = a+b$$

$$(a+b)^0 = 1$$



$$(a+b)^{10} = a^{10} + a^9b + a^8b^2 + a^7b^3 + a^6b^4 + a^5b^5 + a^4b^6 + a^3b^7 + a^2b^8 + ab^9 + b^{10}$$

Combination:-

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

factorial

$2! = 2 \times 1$

$3! = 3 \times 2 \times 1$

$4! = 4 \times 3 \times 2 \times 1 = 4 \times 3!$

$r = 1$ less than the term.

$$r = 1 - 1 = 0$$

$$r = 2 - 1 = 1$$

$${}^{10} C_5 = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 9 \times 4 \times 7 = 252$$

$r=5$
No. of term = $5+1 = 6^{th}$

${}^{10} C_8$
Coefficient of 9th term.

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$n=5$

terms

$1! = (0! = 1)$

$n=5$	terms	
$r=0$	1 st	${}^5C_0 = \frac{5!}{0!(5-0)!} = \frac{5!}{0!5!} = 1$ $1! = (0! = 1)$
$r=1$	2	${}^5C_1 = \frac{5!}{1!(5-1)!} = \frac{5!}{1!4!} = \frac{5 \times 4!}{4!} = 5$
$r=2$	3 rd	${}^5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3!}{2 \times 3!} = 10$
$r=3$	4 th	${}^5C_3 = \frac{5!}{3!2!} = 10$
$r=4$	5 th	${}^5C_4 = \frac{5!}{4!(5-4)!} = \frac{5!}{4! \times 1!} = 5$
$r=5$	6 th	${}^5C_5 = \frac{5!}{5!(5-5)!} = \frac{5!}{5!0!} = 1$

${}^5C_0 = {}^5C_5$, ${}^5C_1 = {}^5C_4$, ${}^5C_2 = {}^5C_3$
 ${}^5C_1 = {}^5C_{5-1}$ ${}^5C_2 = {}^5C_{5-2}$
 ${}^5C_r = {}^5C_{5-r}$

${}^nC_r = {}^nC_{n-r}$

Ex Expand $(2x+1)^4$ in descending powers of x and simplify your answer.

$n=4$ no. of terms = 5

$$(2x+1)^4 = {}^4C_0 (2x)^4 (1)^0 + {}^4C_1 (2x)^3 (1)^1 + {}^4C_2 (2x)^2 (1)^2 + {}^4C_3 (2x)^1 (1)^3 + {}^4C_4 (2x)^0 (1)^4$$

first term = $2x$
 second term = 1

Coefficient:

1 4 6 4 1

${}^4C_0 = \frac{4!}{0! \times 4!} = 1$
 \uparrow
 ${}^nC_r = \frac{n!}{r!(n-r)!}$

Coefficients

1 4 6 4 1

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(2x+1)^4 = \underline{(2x)^4} + 4(2x)^3 + 6(2x)^2 + 4(2x)^1 + \underline{1}$$

$$= 16x^4 + 32x^3 + \underline{24x^2} + 8x + 1$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4 \text{ (ascending)}$$

Ex Expand $(3x-2)^5$ in descending powers of x and simplify your answer.

First term = $3x$

Second term = -2

$$(3x-2)^5 = 243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32$$

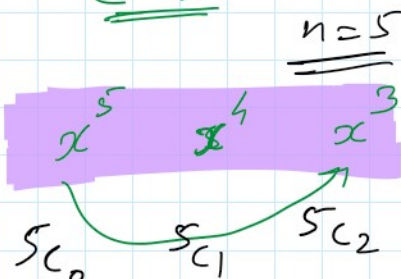
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Ex Consider the expansion of $(x+2)^5$

a) write down the number of terms in this expansion. 6

b) Find the term containing x^3 .

$(x+2)^5$



3rd

x^3

$$n-r=3$$

$$5-r=3$$

$$5-3=r$$

$$\boxed{r=2}$$

3rd term.

$$(a+b)^n = \sum_{r=0}^n {}^n C_r \underline{a^{n-r}} \underline{b^r}$$

Sigma

1st term, $r=0$

$${}^n C_0 a^{n-0} b^0$$

$$= a^n b^0 = \underline{\underline{a^n}}$$

$$\boxed{1-2} \\ 3^{\text{rd}} \text{ term.} \\ {}_5C_2 = \frac{5!}{2!3!} = 10.$$

$$= a^n b^0 \binom{n}{a^n}$$

$$10 \cdot \underline{\underline{(x)^3 (2)^2}}$$

$$\boxed{= 40 x^3}$$

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