Questions

Monday, June 15, 2020 11:00 AM

1) $f(x,y) = \arctan\left(\frac{y}{x}\right)$ **Domain:**

Domain: (x, y)

Domain $\underline{R \times R} = R^2 - \{(x,y) \mid x = 0\}$.

2) $f_x(x,y)$ & $f_y(x,y)$

$f(x,y) = x^3y^2 + x\cos(xy)$

Soln: $f(x,y) = x^3y^2 + x\cos(xy)$

$f_x \Rightarrow$ derivative of $f(x,y)$ w.r.t. x Hold y consta

$$f_x(x,y) = y^2 \frac{\partial}{\partial x}(x^3) + x\frac{\partial}{\partial x}\cos(xy) + \cos(xy)\frac{\partial x}{\partial x}$$

$$= y^2(3x^2) + x(-\sin(xy))\cdot y + \cos(xy) \quad (1)$$

$$= 3x^2y^2 - xy\sin xy + \cos xy$$

$f_y = ?$

$$f_y = x^3\frac{\partial}{\partial y}(y^2) + x\frac{\partial}{\partial y}\cos xy$$

$$= x^3\cdot(2y) + x\cdot(-\sin xy)\frac{\partial}{\partial y}(xy)$$

$$= 2yx^3 - x\sin xy \,. \quad (x)$$

$$= 2yx^3 - x^2\sin xy.$$

Ex'

$f(x,y) = \sqrt{x^2+y^2}$

$$f_x = \frac{x}{\sqrt{x^2+y^2}}, \quad f_y = \frac{y}{\sqrt{x^2+y^2}}$$

Ex:

$$f(x,y,z) = yz\cos(x-z) - xz\sin(y-z) + xy\sin z$$

Find

f_x, f_y, f_z

Soln:

$$f_x = -yz\sin(x-z) - z\sin(y-z) + y\sin z$$

$$f_y = z\cos(x-z) - xz\cos(y-z) + x\sin z$$

$$f_z = y\cos(x-z) + yz\sin(x-z) - [x\sin(y-z) - xz\cos(y-z)] + xy\cos z$$

$$= y\cos(x-z) + yz\sin(x-z) - x\sin(y-z) + xz\cos(y-z) + xy\cos z$$

Ex: $f(x,y) = \arctan\left(\frac{y}{x}\right)$

$f_{xx}, f_{xy}, f_{yy}, f_{yx}$.

$$f_x = \frac{-y}{x^2+y^2}, \quad x \neq 0$$

$$f_y = \frac{1}{x}\frac{1}{1+\frac{y^2}{x^2}} = \frac{x}{x^2+y^2}, \quad x \neq 0.$$

$$f_{xx} = \frac{2xy}{(x^2+y^2)^2}, \quad x \neq 0.$$

$$f_{xy} = \frac{y^2-x^2}{(x^2+y^2)^2}, \quad x \neq 0.$$

$$f_{yy} = \frac{-2xy}{(x^2+y^2)^2}, \quad x \neq 0.$$

$$f_{yx} = \frac{y^2-x^2}{(x^2+y^2)^2}, \quad x \neq 0$$

$f_{xy} = f_{yx}$.

$f_{xx} + f_{yy} = 0$. Harmonic

Ex:

$$u_{xx} + u_{yy} = 0.$$

e^{-x} is continuous in R &
$\cos y$ is also continuous in R
u is continuous in R×R.

Ex

$u_{xx} + u_{yy} = 0.$

cosy is also continuous in
u is continuous in RXR.

$u(x,y) = e^{-x}\cos y.$

$\Rightarrow u_x = -(\cos y)e^{-x}$; $u_{xx} = e^{-x}\cos y$

& $u_y = -e^{-x}\sin y$; $u_{yy} = -e^{-x}\cos y$

So, $u_{xx} + u_{yy} = e^{-x}\cos y - e^{-x}\cos y = \underline{\underline{0}}.$

$\therefore$ $u(x,y)$ is harmonic.

Ex $f(x,y) = xy - x^2 - y^2$

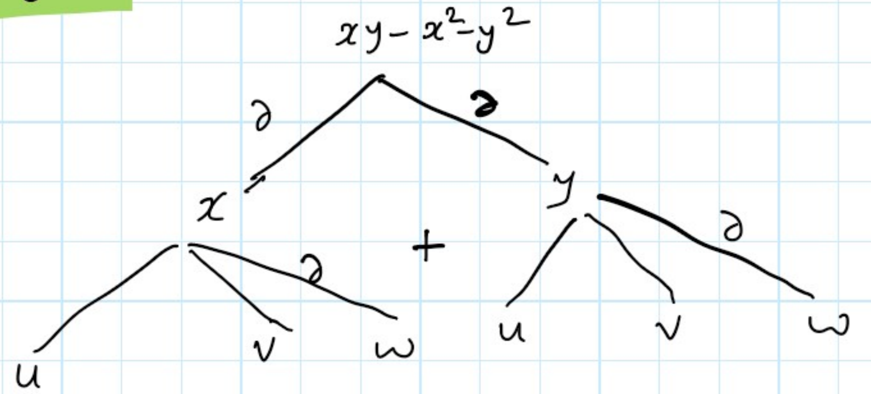
$(1,-2,\pi)$

$x(u,v,w) = ue^v\cos w$, $x(1,-2,\pi) = -1/e^2$

$y(u,v,w) = e^{uv}\sin w$ $y(1,-2,\pi) = 0$

Soln

$f_w = e^{-4}$



$f_w = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial w} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial w}$

$xy - x^2 - y^2$

$x - 0 - 2y$

$x - 2y$

$= \underbrace{(y-2x)(-ue^v\sin w)}_{=0} + (x-2y)e^{uv}\cos w$

$= \left(\frac{-1}{e^2} - 0\right)e^{1(-2)}\cdot\cos\pi = \frac{-1}{e^2} \times e^{-2} \times (-1)$

$= \frac{+1}{e^4}$

Ex $xy - xz + yz = 1.$

$\partial z_x \quad \partial z_y$

$\partial_x z \quad \partial_y z$

$$\begin{cases} \dfrac{\partial z}{\partial x} = -\dfrac{f_x}{f_z} \\ \dfrac{\partial z}{\partial y} = -\dfrac{f_y}{f_z} \end{cases}$$

$$\frac{\partial z}{\partial x} = \frac{y-z}{x-y}$$

$$\frac{\partial z}{\partial y} = \frac{x+z}{x-y}$$

Ex $f(x,y) = 6 - 3x^2 - 2y^2$ at $(x,y)=(1,1)$

$\hat{u}$

$\theta = \pi/3$ with x-axis.

Sol^n

$$D_{\hat{u}} f(1,1) = -3 - 2\sqrt{3}$$

Ex $f(x,y) = \sin(xy)$ at $\left(\sqrt{\pi}/2, \sqrt{\pi}/3\right)$

vector $v = \hat{i} + \hat{j}$ & $w = 3\hat{i} - 4\hat{j}$

Sol^n..

$$D_{\hat{v}} f\left(\sqrt{\pi}/2, \sqrt{\pi}/3\right) = \frac{5\sqrt{6\pi}}{24}$$

$$D_{\hat{w}} f\left(\sqrt{\pi}/2, \sqrt{\pi}/3\right) = -\frac{\sqrt{3\pi}}{10}$$

Gradient =

$\langle y\cos xy, x\cos xy \rangle$

$\langle f_x, f_y \rangle$

$= \left\langle \frac{\sqrt{3\pi}}{6}, \frac{\sqrt{3\pi}}{4} \right\rangle$

$$\hat{v} = \frac{v}{\|v\|} = \frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$$

$$\hat{w} = \frac{w}{\|w\|} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

$$D_{\hat{v}}\left(\frac{\sqrt{\pi}}{2}, \frac{\sqrt{\pi}}{3}\right) = \left\langle \frac{\sqrt{3\pi}}{6}, \frac{\sqrt{3\pi}}{4} \right\rangle \cdot \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

$$=$$

$$D_{\hat{w}}\left(\frac{\sqrt{\pi}}{2}, \frac{\sqrt{\pi}}{3}\right) = \left\langle \frac{\sqrt{3\pi}}{6}, \frac{\sqrt{3\pi}}{4} \right\rangle \cdot \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$$