Revision seq & series

Friday, March 5, 2021 6:03 AM

Tom's parents decide to pay him an allowance each month beginning on his 12th birthday. The allowance is to be £40 for each of the first three months, £42 for each of the next three months and so on, increasing by £2 per month after each three month period.

a Find the total amount that Tom will receive in allowances before his 14th birthday. **(4)**

✓ **b** Show that the total amount, in pounds, that Tom will receive in allowances in the n years after his 12th birthday, where n is a positive integer, is given by $12n(4n + 39)$. **(4)**

(a) 1^{st} yr :- 516 $(40\times3 + 42\times3 + 44\times3 + 46\times3)$

2^{nd} yr :- 612 $(48\times3 + 50\times3 + 52\times3 + 54\times3)$

3^{rd} yr :- 708 $(56\times3 + 58\times3 + 60\times3 + 62\times3)$

$a = 516,\quad d = 612 - 516 = 708 - 612$
$= 96$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2\times516 + (n-1)96\right]$$
$$= n\left[516 + (n-1)48\right]$$
$$= n\left[516 + 48n - 48\right]$$
$$= n\left[468 + 48n\right]$$
$$= 12n\left[4n + 39\right]$$

1) $f(x) = \dfrac{(x-2)}{(1+2x)}$, $x \in R$, $a < x < b$

$$f(x) = (x-2)(1+2x)^{-1}$$
$$= (x-2)(1 - 2x + 4x^2 - 8x^3 \ldots)$$
$$= x - 2x^2 + 4x^3 - 8x^4 - 2 + 4x - 8x^2 + 16x^3$$
$$= -2 + 5x - 10x^2 + 20x^3 - \ldots$$

$(1+2x)^{-1} = {}^{-1}C_0 (1)^{-1}(2x)^0 + {}^{-1}C_1 (1)^{-1-1}(2x)^1 + {}^{-1}C_2 (1)^{-1-2}(2x)^2 + {}^{-1}C_3 (-1)^{-1-3}(2x)^3 + \ldots$

$= 1 + (-1)(2x) + 1(2x)^2 - 1(2x)^3 + \ldots$

$${}^{-1}C_1 = \frac{-1!}{1!\times(-2!)} = \frac{-1\times-2!}{1\times(-2!)} = -1.$$

$\dfrac{n!}{r!\,(n-r)!}$

$${}^{-1}C_2 = \frac{-1!}{2!\times-3!} = \frac{-1\times-2\times\cancel{-3!}}{2\times\cancel{-3!}} = +\frac{2}{2} = 1.$$

$${}^{-1}C_3 = \frac{-1!}{3!\times-4!} = \frac{-1\times\cancel{-2}\times-3\times\cancel{-4!}}{\cancel{2}\times-4!}$$

$$^{-1}C_3 = \frac{-1!}{3! \times -4!} = \frac{-1 \times \cancel{-2 \times -3} \times \cancel{-4!}}{\cancel{3!} \times \cancel{-4!}}$$

$$= -1$$

$$^{-1}C_4 = 1, \quad ^{-1}C_5 = -1$$

$$\boxed{r = -2x} = \frac{-10x^2}{5x}$$

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1$$

$$f(x) = -2 + (5x - 10x^2 + 20x^3 + \dots)$$

$$f(x) = \left(\frac{x-2}{1+2x}\right) = \frac{(x-2)}{1-(-2x)} = \frac{a}{1-r} = S_\infty$$

$r = -2x.$

$|-2x| < 1$

$|2x| < 1$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

$a = -\frac{1}{2}, \quad a = \frac{1}{2}$

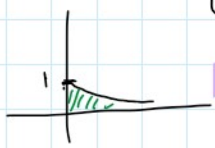
$= 2^0 + 2^1 + 2^2 + 2^3 + \dots$

G.P., $r = 2$

$1 + 2 + 4 + 8 + 16 + \dots$

$\Rightarrow \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} - \dots$

GP. $\boxed{r = \frac{1}{2}}$



$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

The series expansion of $(1+ax)^b$ up to and including the term in x^2 is given by $1-\frac{3}{2}x-\frac{3}{8}x^2$. Find the values of a and b. Explain why the substitution $x=-2$ may not be suitable in estimating the value of $5^{\frac{7}{4}}$ using the above series. [5]

$$(1+ax)^b = 1 + b(ax) + \frac{b!}{2!(b-2)!}(ax)^2 + \dots$$

$$= 1 + abx + \frac{b \times (b-1) \times \cancel{(b-2)!}}{2 \cdot \cancel{(b-2)!}} a^2 \cdot x^2$$

$$= 1 + abx + \frac{b(b-1)}{2} a^2 x^2 = 1 - \frac{3}{2}x - \frac{3}{8}x^2$$

$$ab = -\frac{3}{2} \quad \& \quad \frac{b(b-1)}{2} a^2 = -\frac{3}{8}$$

$$\Rightarrow \frac{a^2b^2 - a^2b}{2} = -\frac{3}{8}$$

$$\Rightarrow (ab)^2 - a(ab) = -\frac{3}{4}$$

$$\frac{9}{4} + \frac{3a}{2} = -\frac{3}{4}$$

$$\frac{9}{4} + \frac{3a}{2} = -\frac{3}{4}$$

$\boxed{b = \frac{3}{4}}$

$$\Rightarrow \frac{3a}{2} = -\frac{3}{4} - \frac{9}{4} = -\frac{12}{4} = -3$$

$$\Rightarrow \boxed{a = -2}$$

$x = -2$

$$[1 + (-2)x]^{3/4} = 1 - \frac{3}{2}x - \frac{3}{8}x^2$$

$x = -2$

$$[1 + (-2)(-2)]^{3/4} = [1+4]^{3/4}$$
$$= [5]^{3/4}$$

$\boxed{5^{3/4}}$ ✗

Q

$a_7 = 192$, $S_2 = -3$

GP, $S_6 = ?$

$$a_n = a_1 r^{n-1}$$

$$a_7 = a_1 r^{7-1} = a_1 r^6$$

$$\boxed{192 = a_1 r^6}$$

$$a_1 + a_1 r = -3$$

$$a_1(1+r) = -3$$

$$\boxed{a_1 = \frac{-3}{1+r}}$$

$$192 = \frac{-3}{1+r} \cdot r^6$$

$$\cancel{192}(1+r) = -\cancel{3}r^6$$

$$-64 - 64r = r^6$$

$$r^6 + 64r + 64 = 0$$

$$\boxed{r = -2}$$

$r = \pm 1, \pm 2$

$$(-2)^6 - 128 + 64$$
$$64 - 128 + 64 = 0$$

$$a_1 = \frac{-3}{1-2}$$
$$= \frac{-3}{-1} = 3$$

$$\boxed{a_1 = 3}$$

$$\boxed{S_6 = \frac{a(1-r^n)}{1-r}}$$

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7. A school is collecting non-perishable food items for a food drive. To get students interested/invested in the drive, they are going to attempt to build the largest triangular pyramid of cans possible. The rows are built according to the pattern: 1, 3, 6, 10, 15, ...

   a. Show that the number of cans in the n^th is given by: $u_n = \frac{1}{2}n^2 + \frac{1}{2}n$. [4]

   b. Hence, determine how many cans are in the 10th row the pyramid. [1]

7. A school is collecting non-perishable food items for a food drive. To get students interested/invested in the drive, they are going to attempt to build the largest triangular pyramid of cans possible. The rows are built according to the pattern: 1, 3, 6, 10, 15, ...

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b. Hence, determine how many cans are in the 10th row the pyramid. [1]

c. Hence also, determine the number of rows of a pyramid with a bottom that is made up of 325 cans. [2]

Q 1, 3, 6, 10, 15, + - - -.

Triangular series

$\boxed{u_n = \frac{1}{2}n^2 + \frac{1}{2}n}$

$= \frac{1}{2} n(n+1)$

1
3
6
10
15

$a_1 = 1$
$a_2 = 3$

$n=1 \quad 1 = 1$

$n=2 \quad 3 = 1 + 2$

$n=3 \quad 6 = 1 + 2 + 3$

$n=4 \quad 10 = 1 + 2 + 3 + 4$

$n=5 \quad 15 = 1 + 2 + 3 + 4 + 5$

⋮

so for n^{th} term $= u_n = (1 + 2 + 3 + 4 + 5 + ---- + n)$

$= {*}\left(\frac{n(n+1)}{2}\right) = \frac{1}{2}n^2 + \frac{1}{2}n$

AP.

$= \frac{n}{2}[2(1) + (n-1)1]$

$= \frac{n}{2}[2 + n - 1] = \left(\frac{n(n-1)}{2}\right)^{*}$