Partial Derivative

Sunday, April 12, 2020 8:58 PM

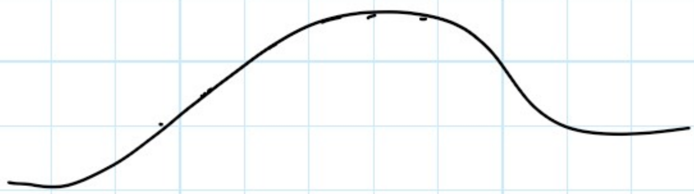
$y = f(x)$

Dependent variable → y

Independent :- x

$\frac{dy}{dx} = ?$

$\frac{\partial y}{\partial x}$



$Z = f(x, y)$

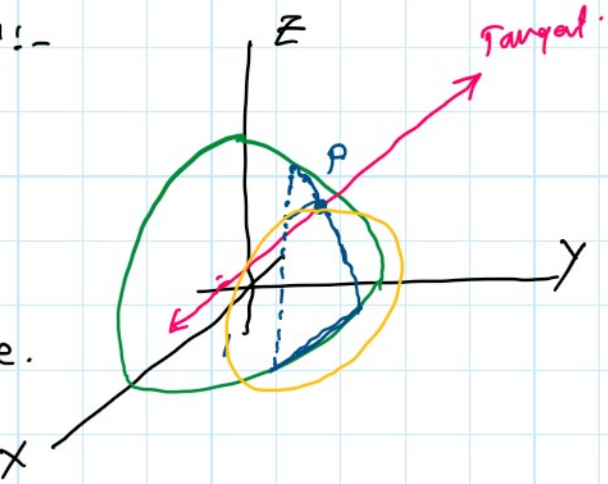
Independent variable 2 (x & y)

dependent variable is Z

$Z = \sqrt{x^2 + y^2 - 4}$ 3D

Derivative of multivariable f^n :-

Q what does the derivative of multivariable f^n look like?



$f(x, y) = Z$ is 3-D.

It is a slope of tangent to surface at a point. (∞ tangents)

→ To find ~~the~~ slope of the tangent line to a surface at a point, we must give a Tangent line a direction.

→ Fully directional derivative come later

For now, we restrict our derivative to

In the x-direction or

In the Y-direction.

→ To find the slope of tangent line in x-direction

- we must contain the tangent line in a plane parallel to the XZ plane (contain X-axis).

→ Requires "y" to be hold constant. (eg. If $y = 3$, plane $\parallel XZ$)

→ For y-direction: Hold "x" constant forces tangent line to be in a plane $\parallel$ to XZ plane.

$$f(x, y) = 2x^2y^3$$

The idea of treating a variable as a constant & thereby insuring that the tangent line is in the direction of the other variable is called a partial derivative

$$\frac{\partial y}{\partial x}.$$

Notation for $f(x, y) = Z$

$\frac{\partial f}{\partial X}$, 'Holds' Y constant. (plane $\parallel XZ$) so it gives slope of the tangent line to the surface at a point in X-direction. (w.r.t x)

$$\left.\begin{matrix}\frac{\partial f}{\partial X}, \frac{\partial Z}{\partial X}, f_x, Z_x.\end{matrix}\right\} \text{ w.r.t } x$$

$$\left.\begin{matrix}\frac{\partial f}{\partial y}, \frac{\partial Z}{\partial y}, f_y, Z_y\end{matrix}\right\} \text{ w.r.t } y.$$

Q. $f(x, y) = 2x^2y^3 - 3x^2y + 2x^2 + 3y^2 + 1.$

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Manav $\frac{\partial f}{\partial x} = 4xy^3 - 6xy + 4x.$ ✓

Dev $\frac{\partial f}{\partial y} = 6x^2y^2 - 3x^2 + 6y$ V.G.

Q) $f(x,y) = e^x \cos y + e^y \sin x.$

$f_x = e^x \cos y + e^y \cos x.$

$f_y = -e^x \sin y + e^y \sin x.$

Q $z = xe^{xy^2}$

M $z_x = e^{xy^2}(1 + xy^2)$ ✓

D. $z_y = 2x^2 y e^{xy^2}$

$z = xe^{xy^2}$ (Hold y as constant)

$z_x = x \frac{\partial}{\partial x} e^{xy^2} + e^{xy^2} \frac{\partial}{\partial x}[x]$

$= x \cdot e^{xy^2} \frac{\partial}{\partial x}[xy^2] + e^{xy^2} \cdot (1)$

$= x \cdot e^{xy^2} \cdot y^2 + e^{xy^2}$

$= e^{xy^2}[1 + xy^2]$

Implicit:

z is the implicit defined variable ("Not Y")

Q. $x^2y + xz + yz^2 = 8$ [Both side]

D) z_x $2xy + xz_x + z(1) + 2yzz_x = 0$

z_x $\quad 2xy + x z_x + z(1) + y\,2z z_x = 0$

M

z_y

$$z_x(x + 2yz) = -2xy - z$$

$$z_x = \frac{-2xy - z}{x + 2yz}$$

$$\underline{z_y} = \frac{-x^2 - z^2}{x + \underline{2yz}}$$

$$x^2y + xz + yz^2 = 8$$

$$x^2\left(\frac{\partial y}{\partial y}\right) + x\frac{\partial z}{\partial y} + y\frac{\partial z^2}{\partial y} + z^2\cdot\left(\frac{\partial y}{\partial y}\right) = 0$$

$$x^2 + x z_y + y\cdot 2z z_y + z^2 = 0.$$

$$z_y(x + 2yz) = -x^2 - z^2$$

$$z_y = \frac{-x^2 - z^2}{x + 2yz}$$

Higher Derivative:-

$$Z = f(x,y)$$

$$\frac{\partial f}{\partial x}$$

$$\frac{\partial^2 f}{\partial x \partial x} \Rightarrow f_{xx},\ Z_{xx}$$

$$\boxed{\frac{\partial^2 f}{\partial y \partial x}} \Rightarrow f_{xy},\ Z_{xy}$$

$$\frac{\partial f}{\partial y}$$

$$\frac{\partial^2 f}{\partial y^2} \Rightarrow f_{yy},\ Z_{yy}$$

$$\frac{\partial^2 f}{\partial x \partial y} \Rightarrow f_{yx},\ Z_{yx}.$$

Q $f(x,y) = x\sin^2 y + y^2\cos x.$, $\underset{\text{Mar}}{\underline{f_{xy}}}$ & $\underset{\text{Dev.}}{\underline{f_{yx}}} = ?$

$$f_x = \sin^2 y - y^2\sin x.$$

$$f_{xy} = 2\sin y\cdot\cos y - 2y\sin x$$

Ohh both are same.

$f_x = \sin y - y \ldots$

$f_{xy} = 2\sin y \cdot \cos y - 2y\sin x$

Ohh both are same.

Mixed derivative is always equal.

$f_y = 2x\sin y\cos y + 2y\cos x.$

$f_{yx} = 2\sin y\cos y - 2y\sin x.$

$f_{xy} = f_{yx} \neq f_{xx}$

$\neq f_{yy}$

The function is continuous on a region, mixed partial derivative are equal.

$f_{xy} = f_{yx}.$

$w = f(x,y,z)$

$g_{xyz} = g_{yxz} = g_{zxy}$

$\neq g_{xyx}$

$\neq g_{xzz}$

$g_{xyy} \neq g_{yyx}$